Estimating the Background Potential of Quantum Constrictions Using Scanning Gate Microscopy and Machine Learning

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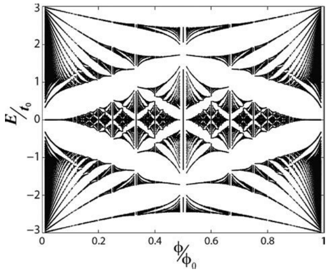
65th Electronic Materials Conference
Nanoscale Characterization – June 28th

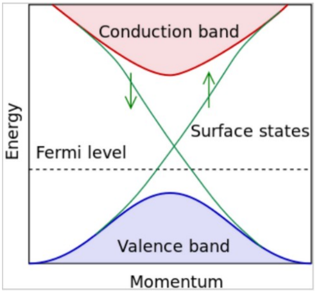
Quantum Materials

Quantum Hall

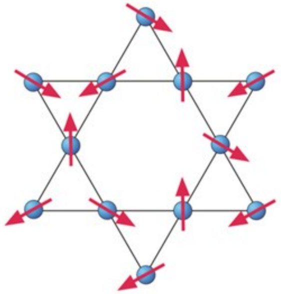


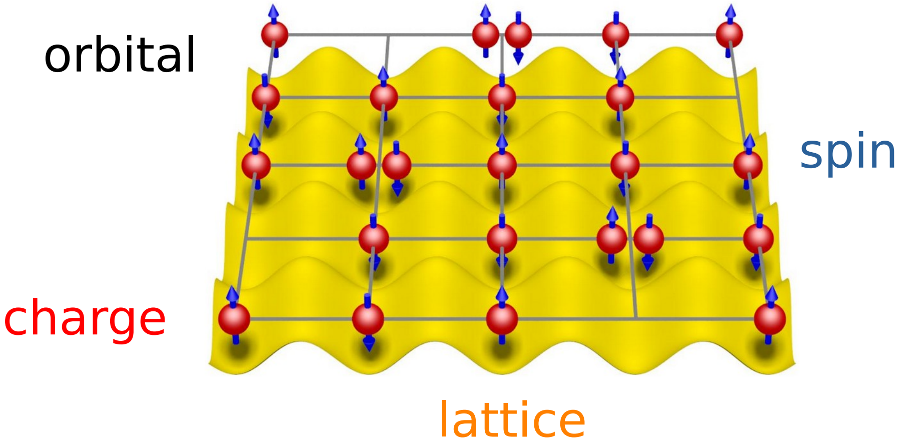
Superconductivity

Topological insulators



Spin liquids

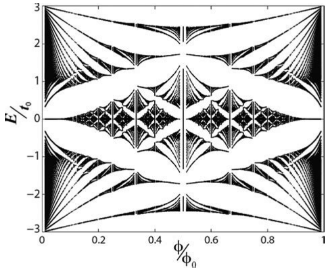




(*) Images obtained from the internet.

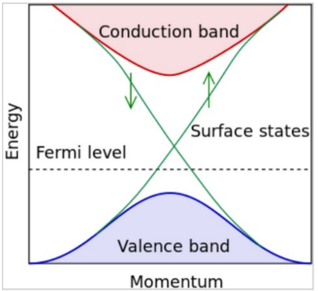
Quantum Materials

Quantum Hall

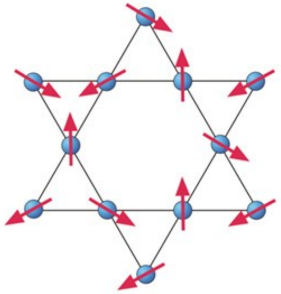


Superconductivity

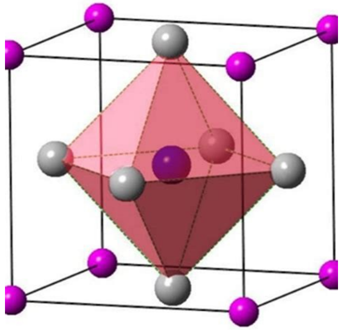
Topological insulators

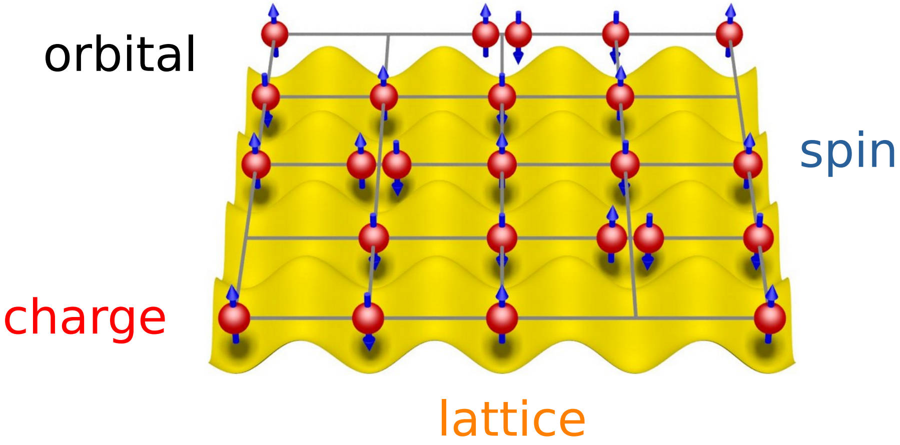


Spin liquids

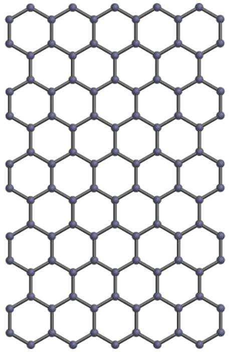


Bi_2Te_3





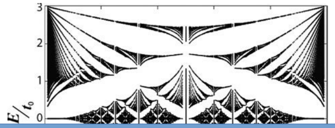
Graphene



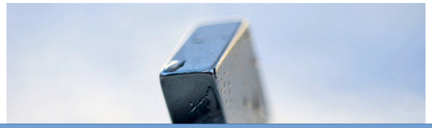
(*) Images obtained from the internet.

Quantum Materials

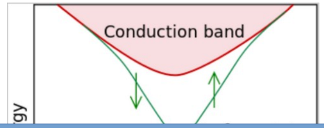
Quantum Hall



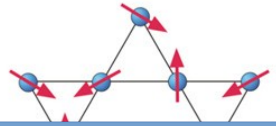
Superconductivity



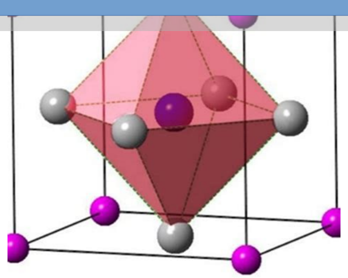
Topological insulators

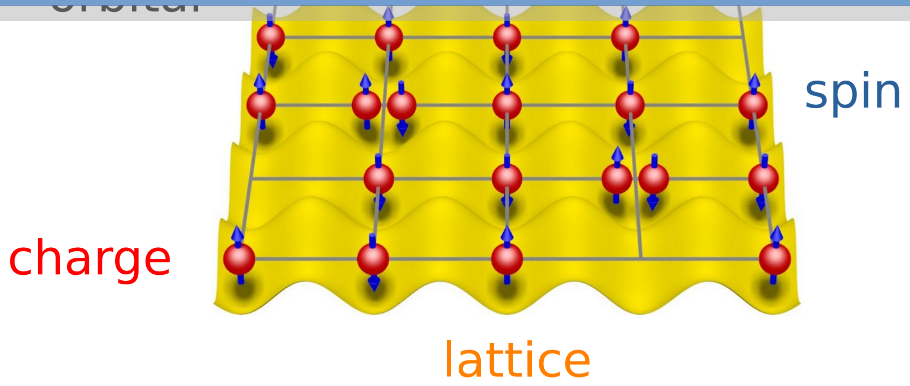


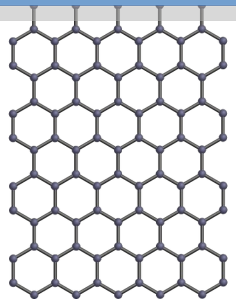
Spin liquids



Defects play a major role!





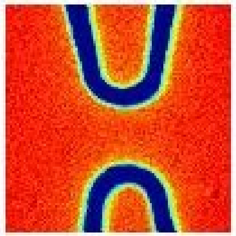


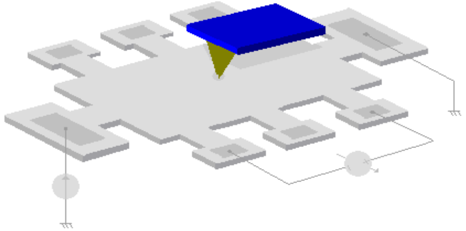
(*) Images obtained from the internet.

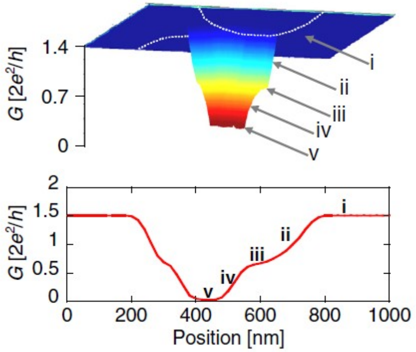
Structure of the Talk

- Quantum Point Contacts (QPC)
- Scanning Gate Microscopy (SGM)
- Device Fabrication
- Characterization
  - Transmission
  - Shubnikov-de Haas
  - SGM
- Machine Learning
  - Convolutional Neural Nets
  - Cellular Neural Nets
- Stigmergy Meta-Heuristics
  - Roughness
  - Correlations

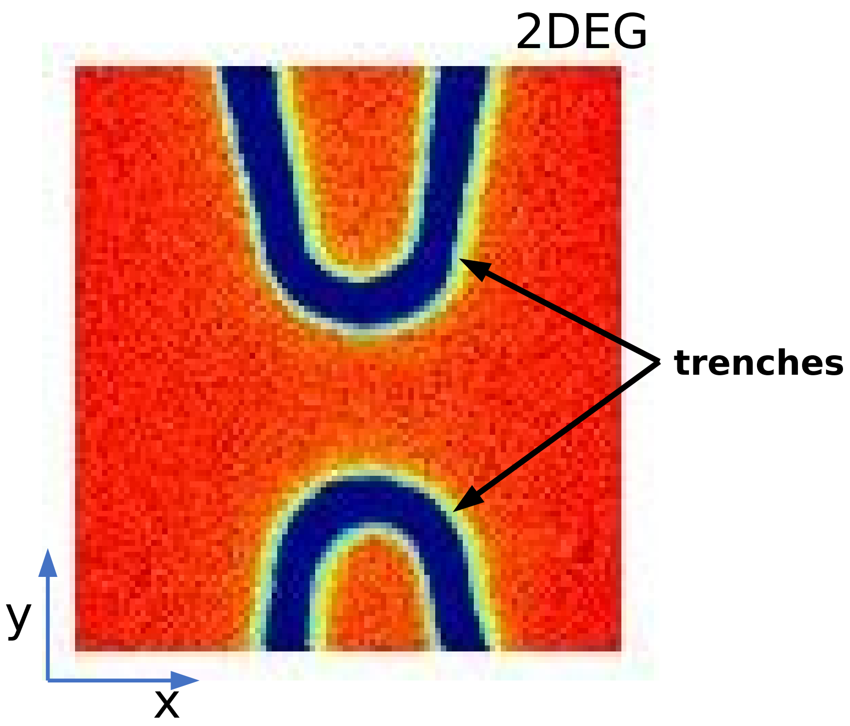
- Conclusions

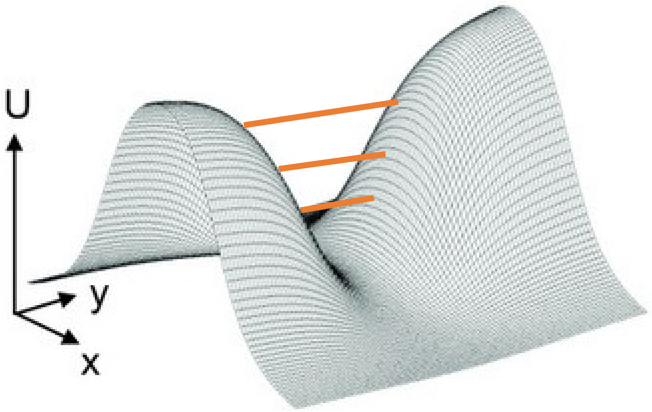


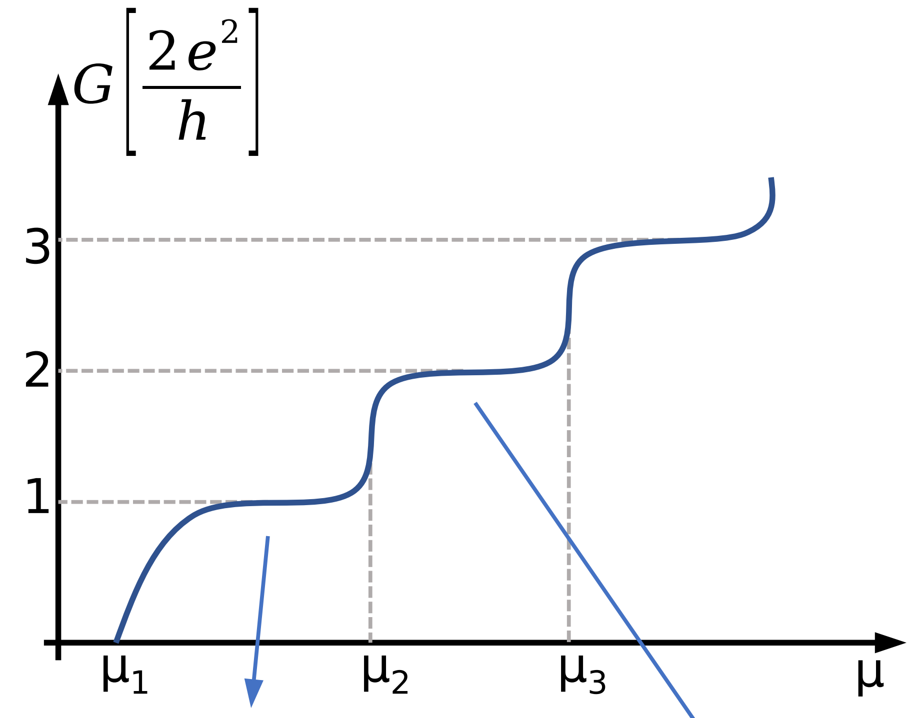


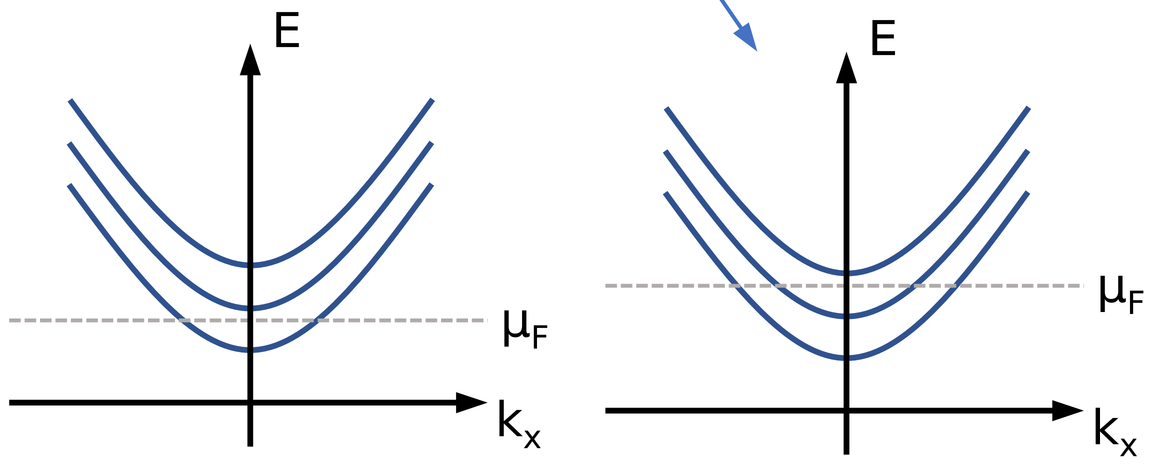


Quantum Point Contacts







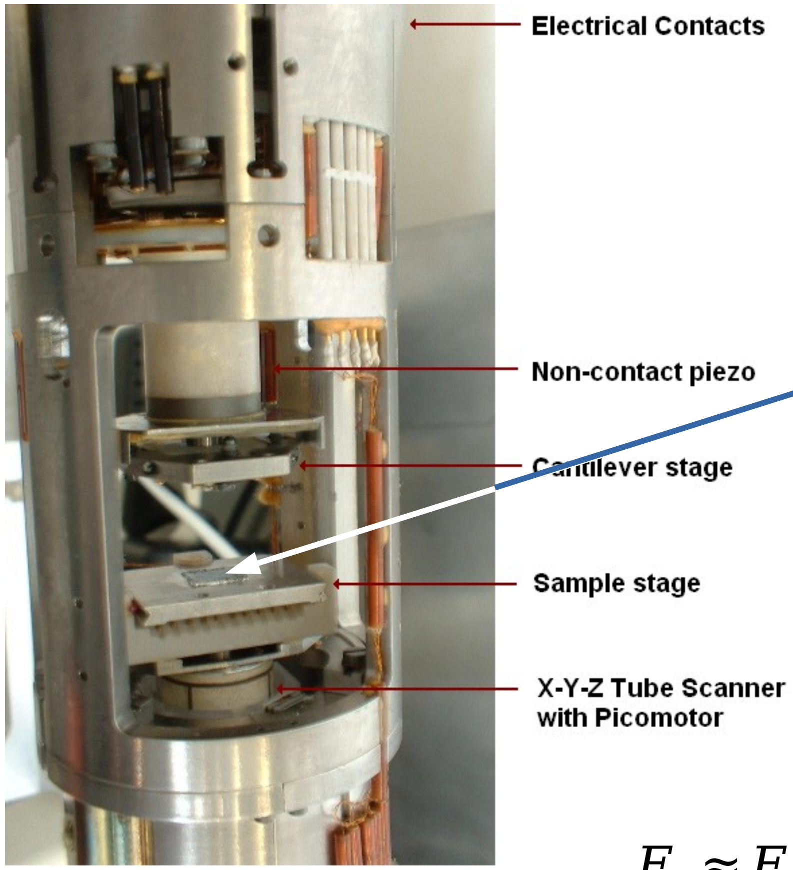


Bart Van Wees
(U. Gronigen – Netherlands)

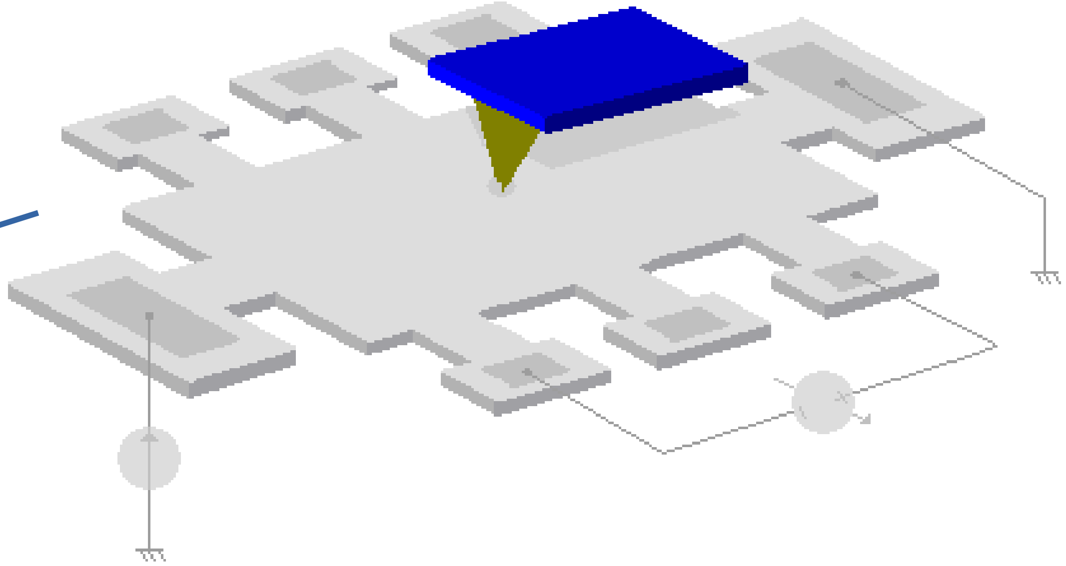
Quantum sensor

B.J. van Wees et al. "Quantized conductance of point contacts in a two-dimensional electron gas". *Phys. Rev. Let.* **60**-9 (1988) 848–850.

1st Order Perturbation Theory:



$$V = V_0 \delta(\mathbf{r} - \mathbf{r_0})$$



$$E_n \approx E_n^o + \langle n | V | n \rangle$$

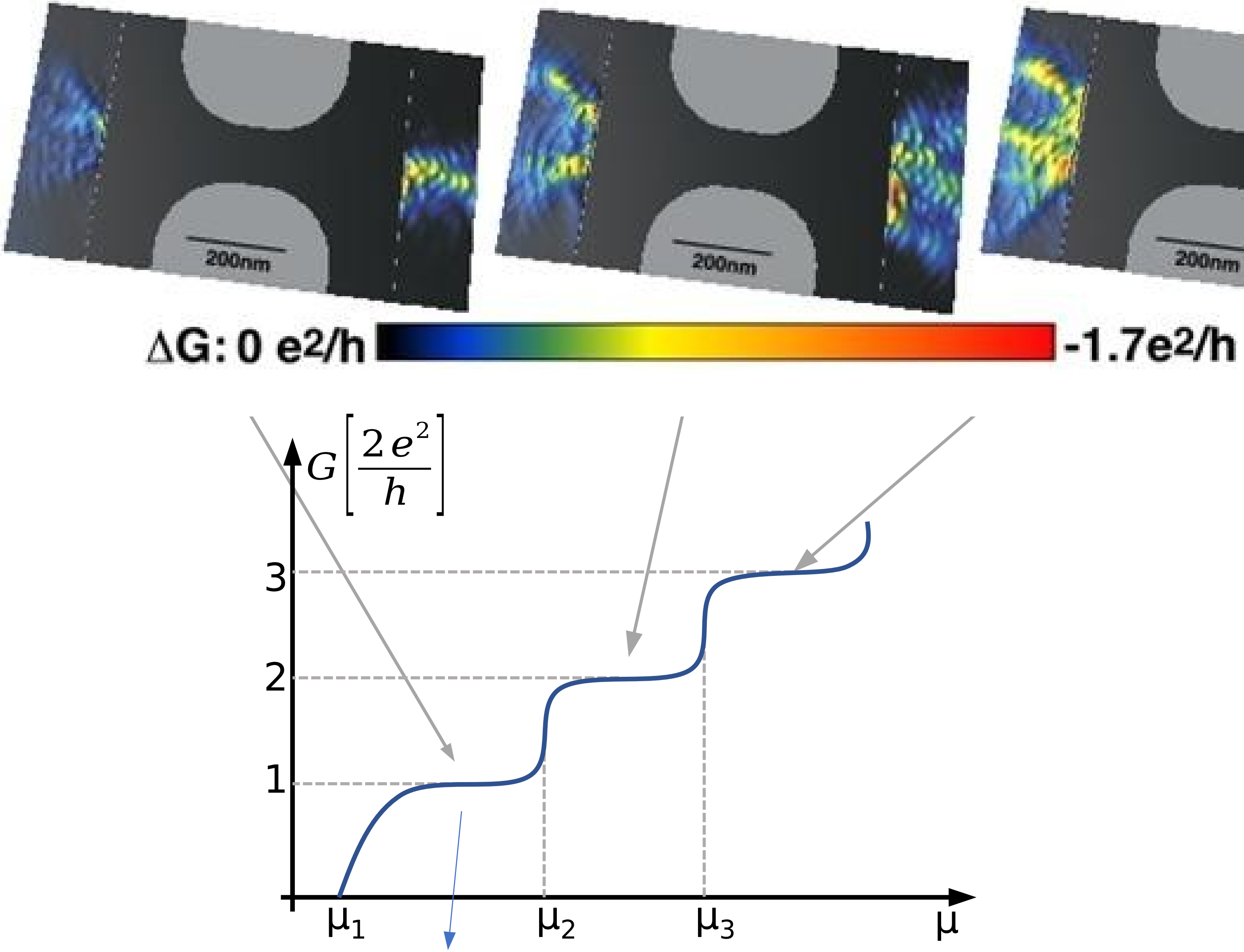
$$\Delta E_n \approx V_0 |\varphi_n(\mathbf{r_0})|^2$$

$$\Delta G(\mathbf{r_0}) \approx \frac{\partial G}{\partial E_F} \Delta E_n$$

$$\Delta G(\mathbf{r_0}) \approx V_0 \frac{\partial G}{\partial E_F} |\varphi_n(\mathbf{r_n})|^2$$

Shifts proportional to LDOS

Electron Flow

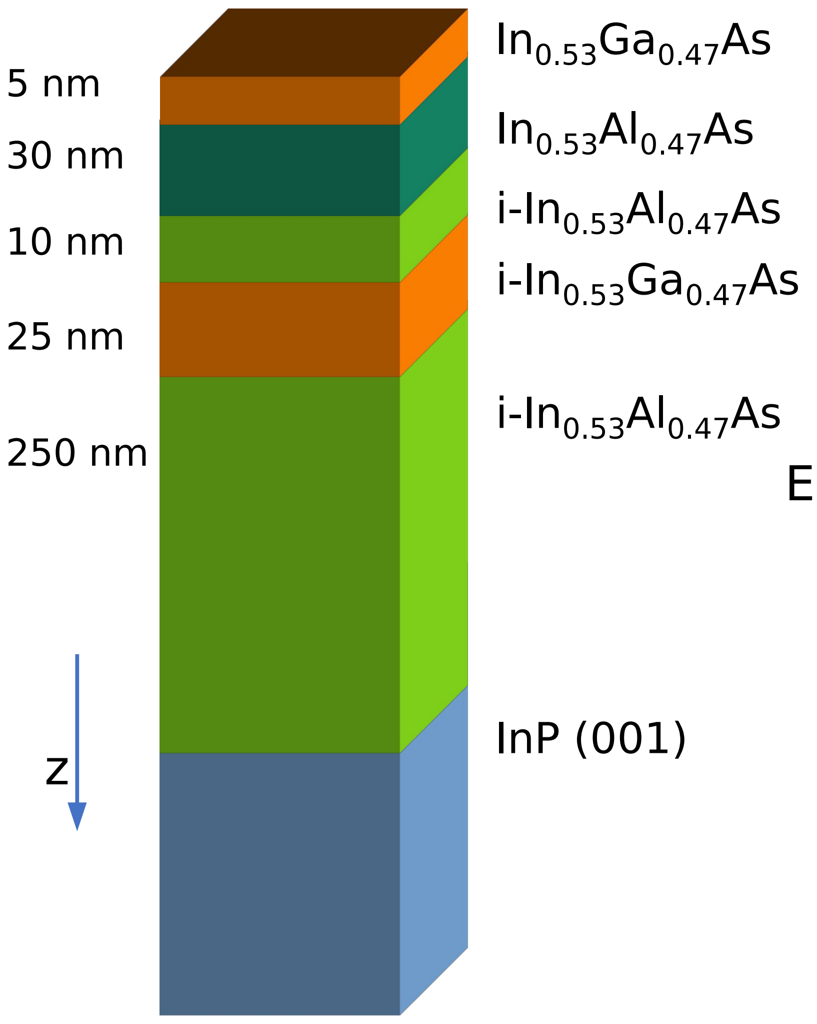


R. Westervelt
(Harvard)

M. A. Topinka, B. J. LeRoy, S. E. J. Shaw, E. J. Heller, R. M. Westervelt, K. D. Maranowski, A. C. Gossard, “Imaging Coherent Electron Flow from a Quantum Point Contact”, *Science* **289** (2000) 2323.

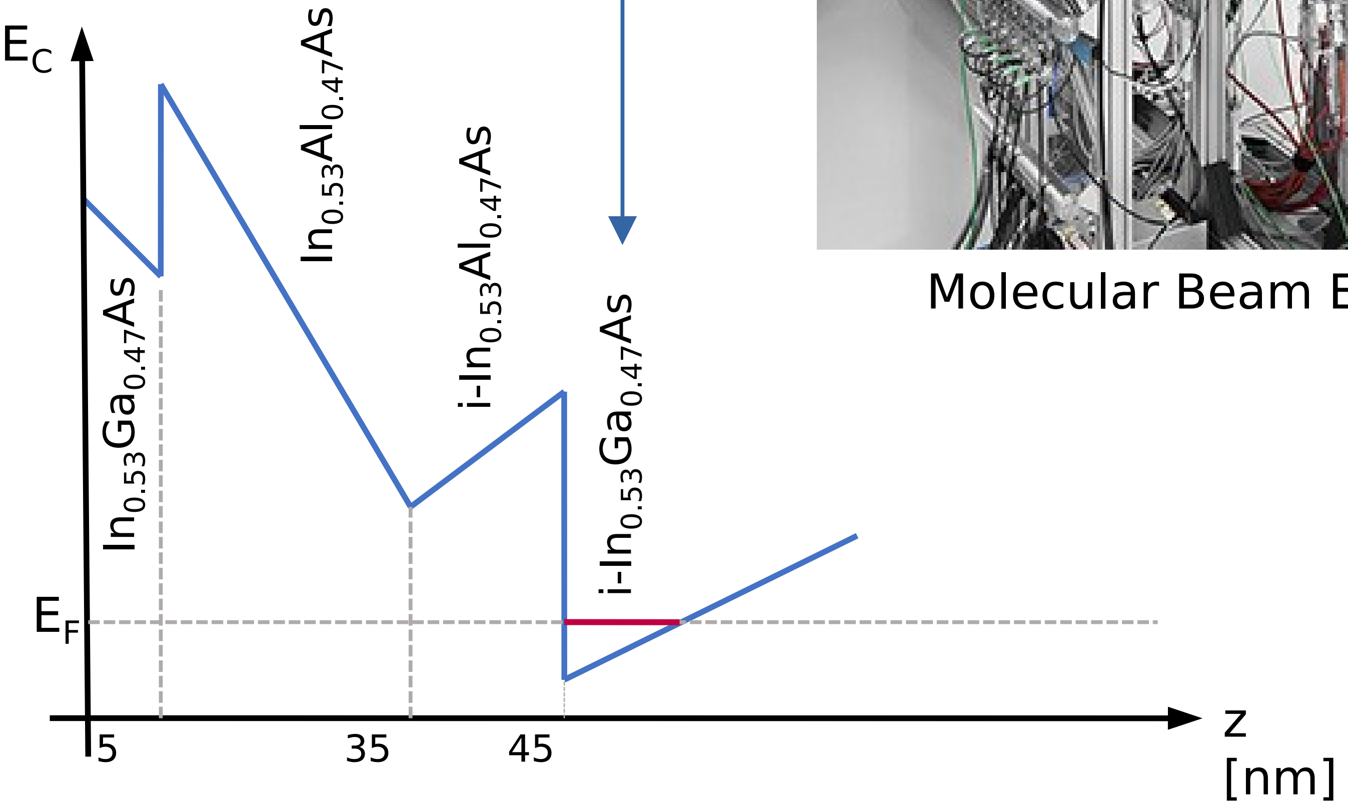
Device Fabrication

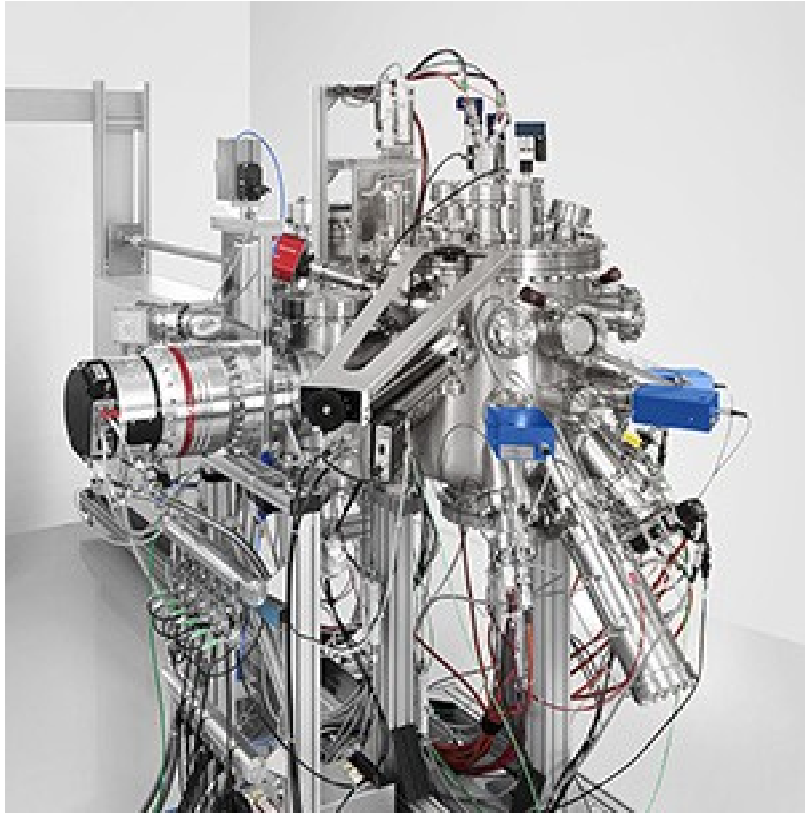
Heterostructure



MBE-grown quantum well

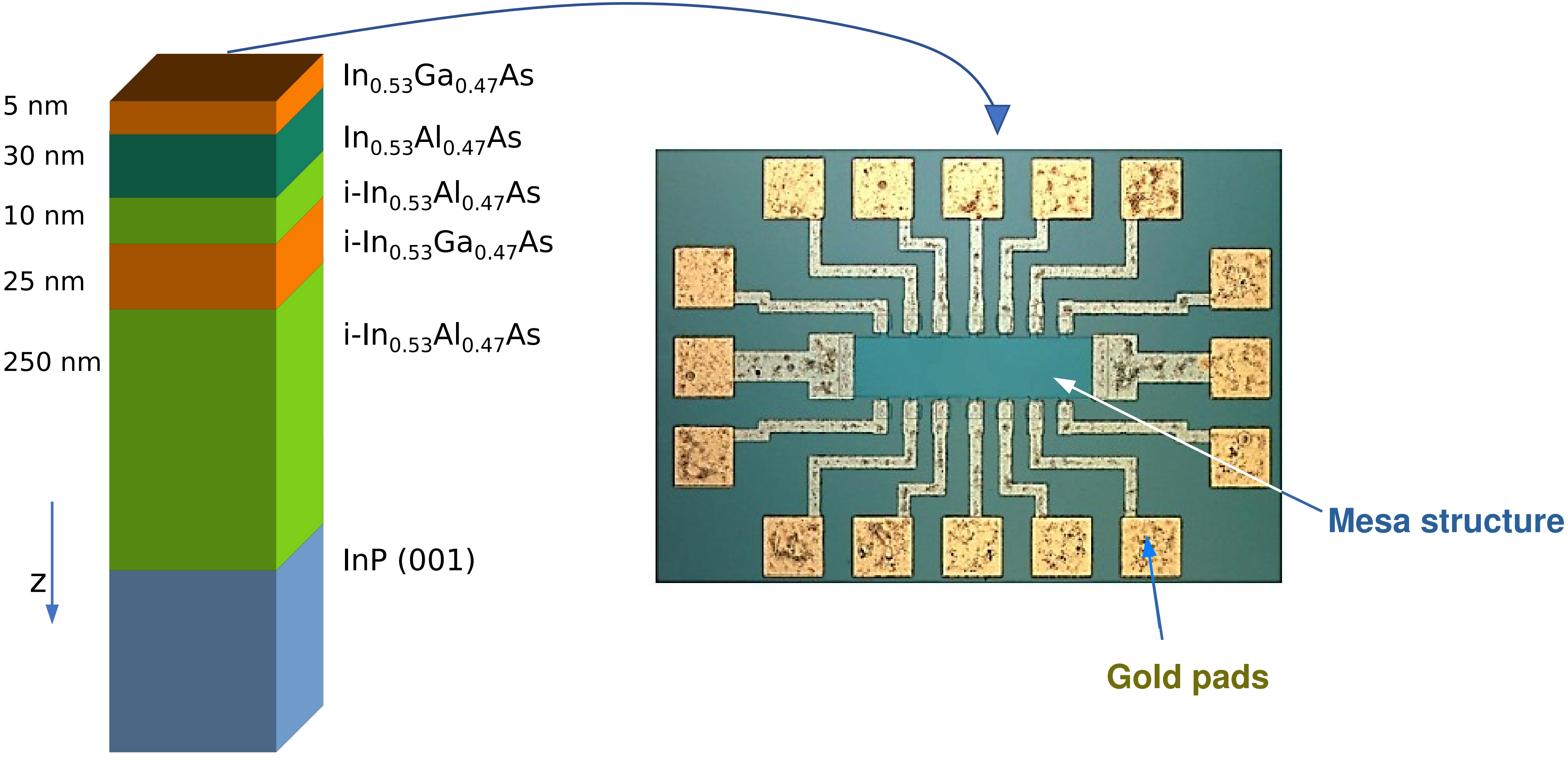
2DEG





Molecular Beam Epitaxy

Hall bar



5 nm
30 nm
10 nm
25 nm
250 nm
z

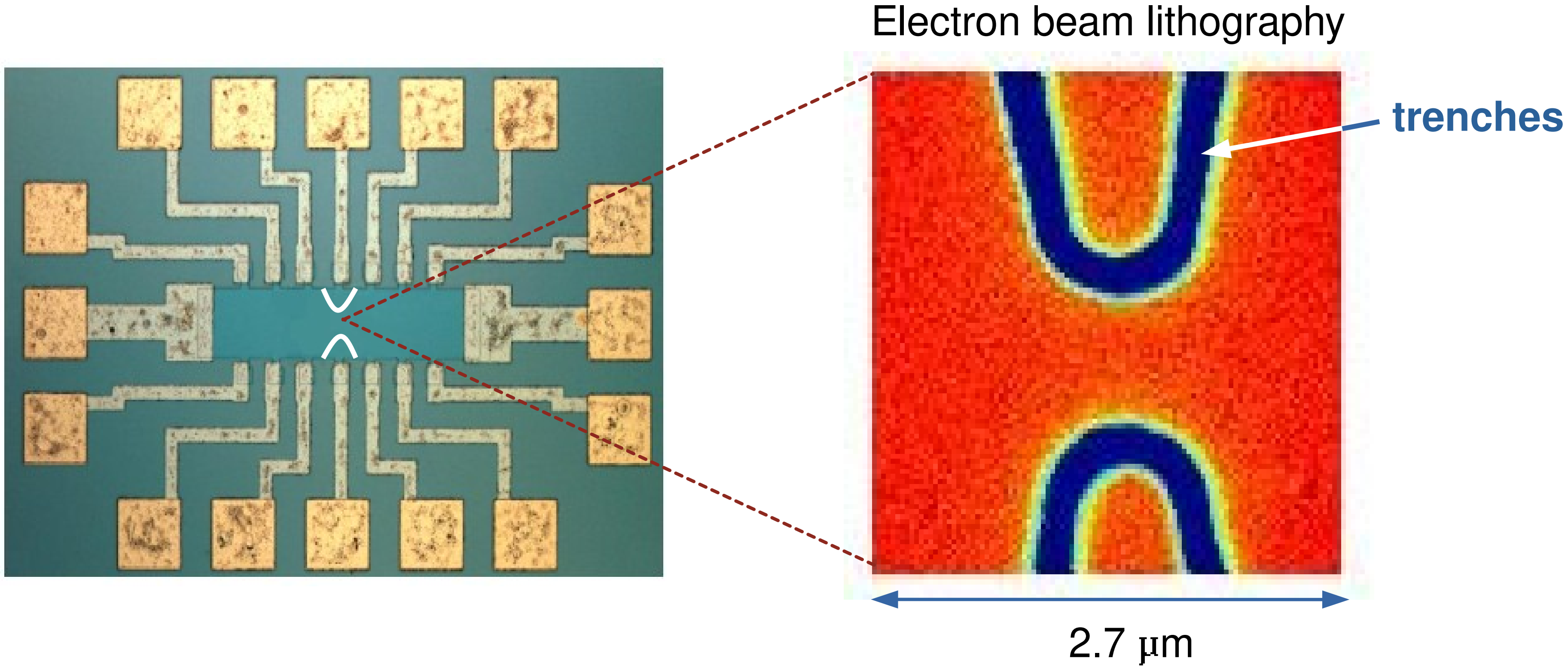
$In_{0.53}Ga_{0.47}As$
$In_{0.53}Al_{0.47}As$
$i\text{-}In_{0.53}Al_{0.47}As$
$i\text{-}In_{0.53}Ga_{0.47}As$
$i\text{-}In_{0.53}Al_{0.47}As$
InP (001)

Mesa structure

Gold pads

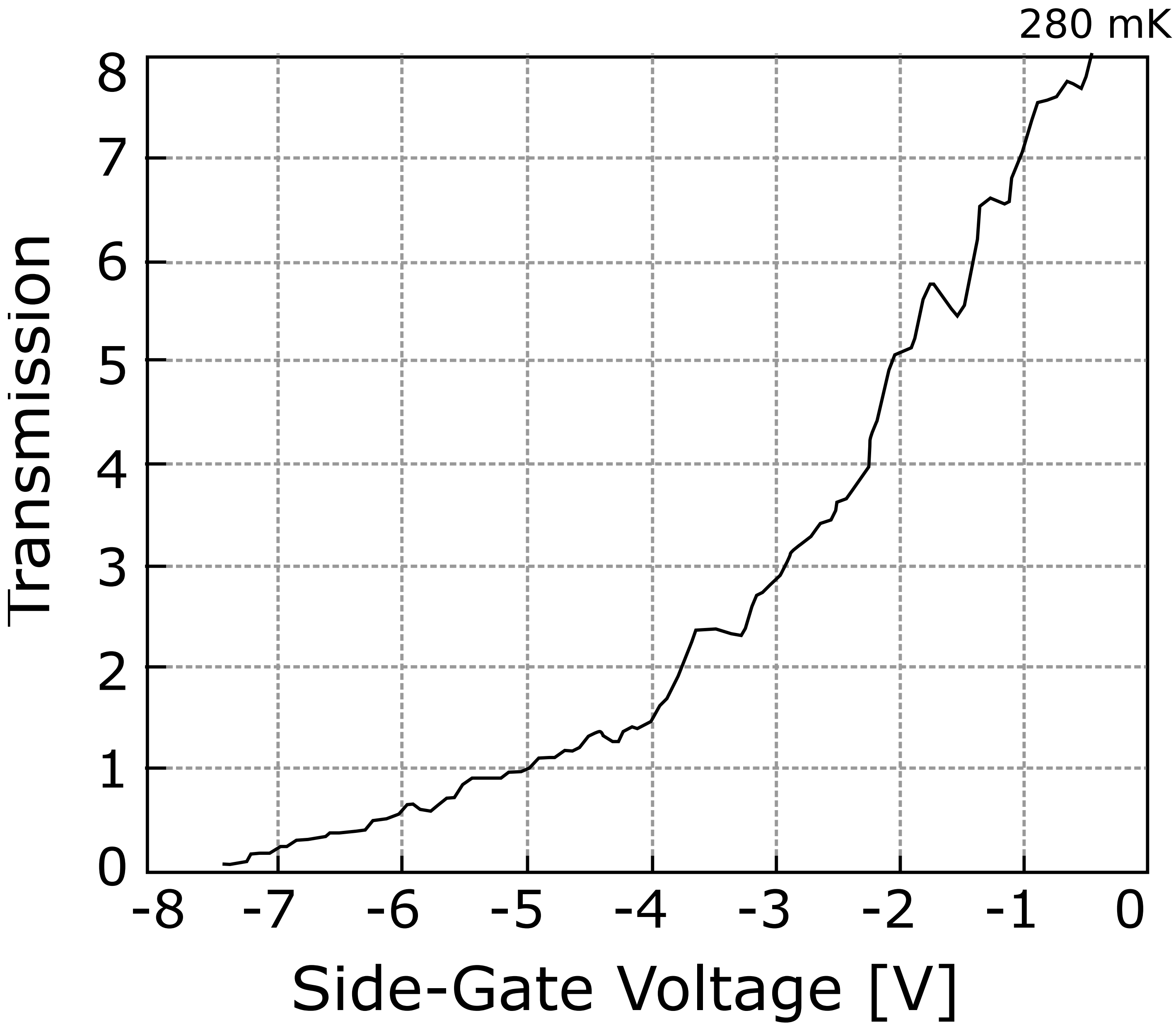
MBE-grown quantum well

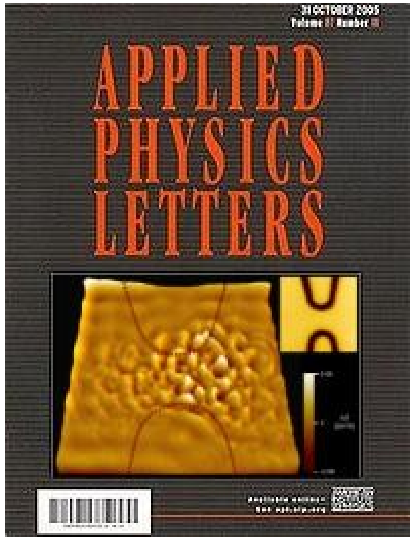
EBL-Defined QPC



Characterization

- Shubnikov de-Haas
- Quantum Hall
- Quantum Chaos

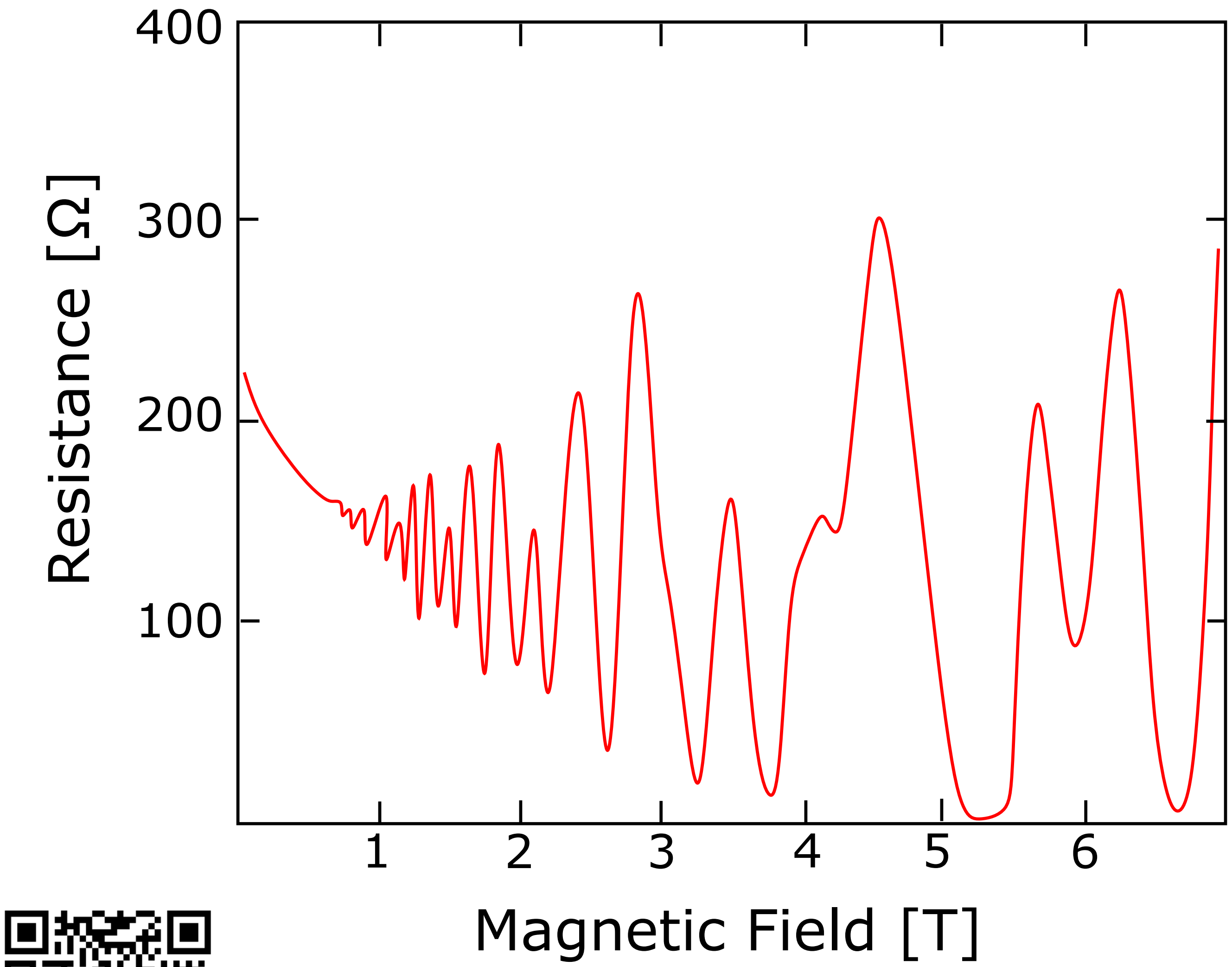



31 OCTOBER 2005

APPLIED PHYSICS LETTERS

C. R. da Cunha, et al.
Imaging of quantum interference patterns within a quantum point contact
Appl. Phys. Lett. **89-24** (2006) 242109.

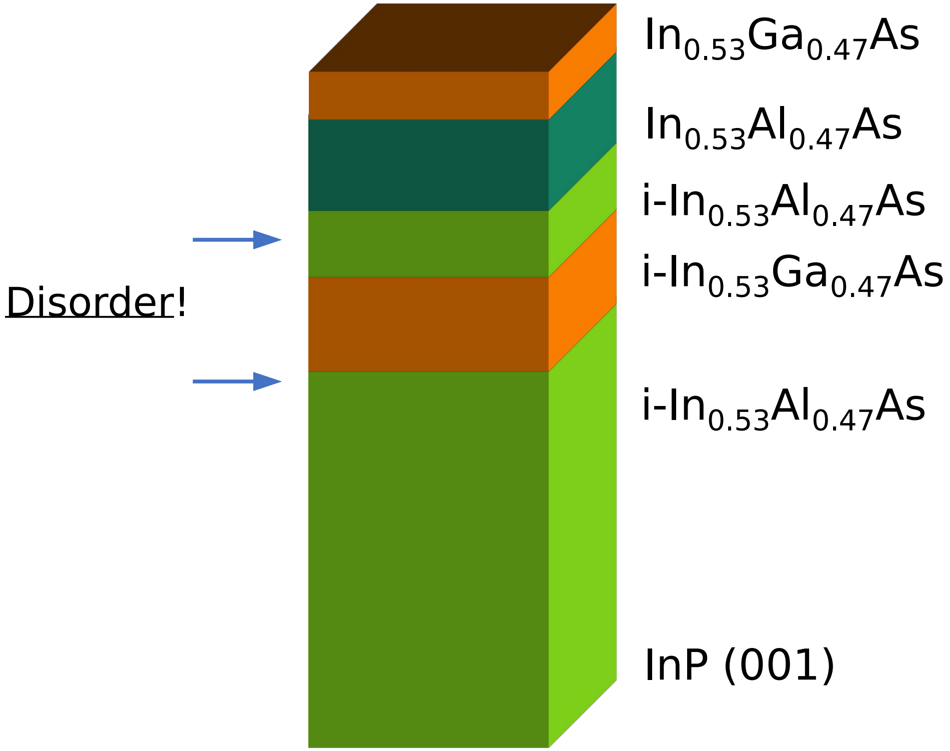
Shubnikov-de Haas



$n = 7.2 \times 10^{11}\ \mathrm{cm}^{-2}, 2.1 \times 10^{11} \mathrm{cm}^{-2}$

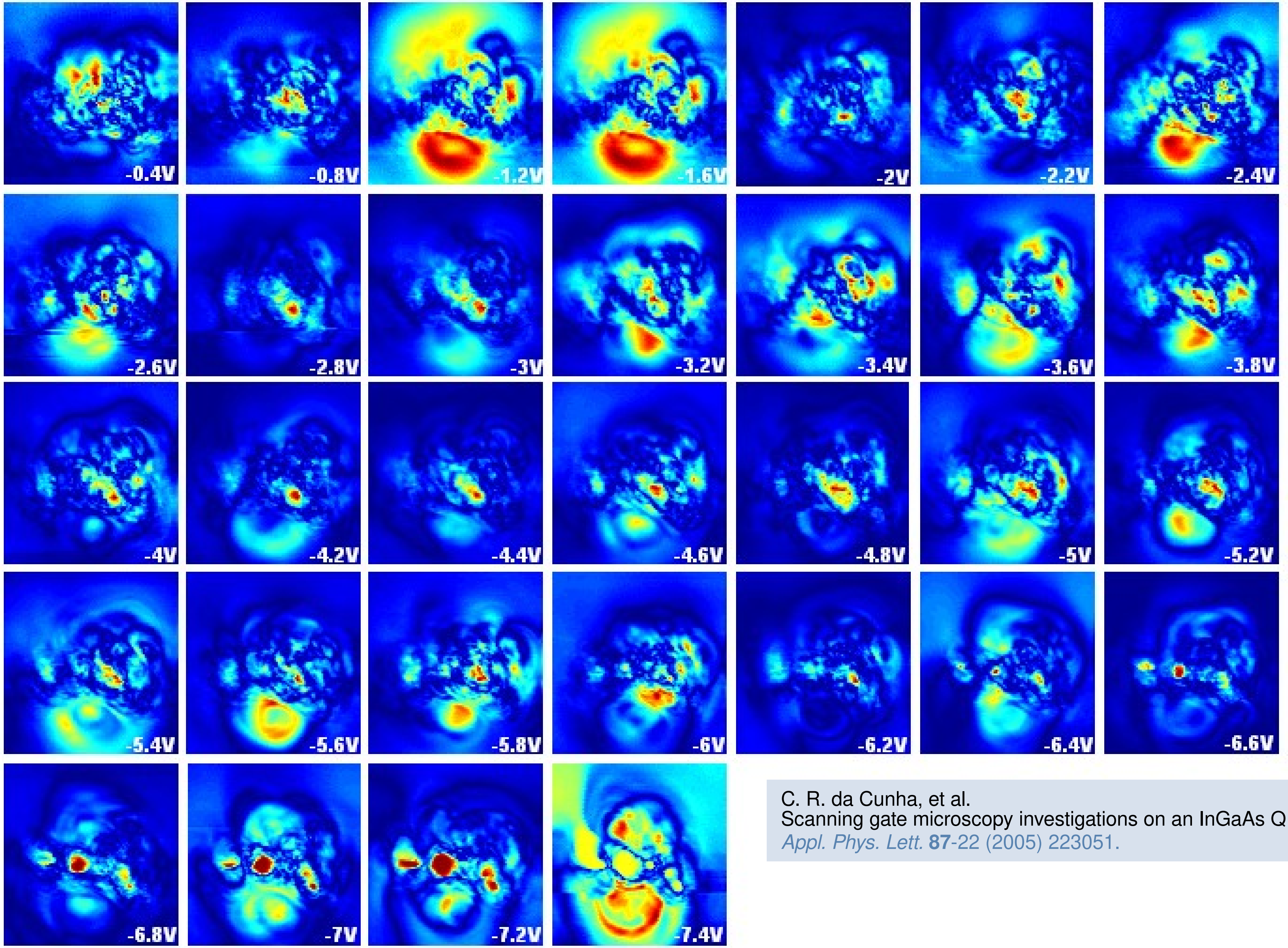
$\mu = 7.4 \times 10^{4}\ \mathrm{cm}^{2}/\mathrm{V.s}$

$l = 1.2\ \mu\mathrm{m}$



C. R. da Cunha, et al.
Scanning gate imaging of a disordered quantum point contact
J. Phys. Cond. Matt. **26-19** (2014) 193202.

Scanning Gate Microscopy



C. R. da Cunha, et al.
Scanning gate microscopy investigations on an InGaAs QPC
Appl. Phys. Lett. **87**-22 (2005) 223051.

Too Many Assumptions...

IF {

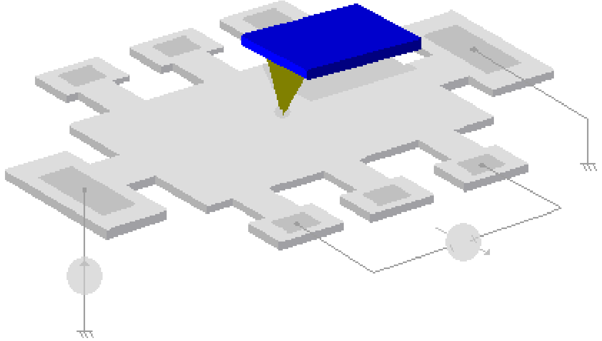
1. The perturbation is sufficiently small;
2. The induced potential is delta-shaped
3. Wave function is given solely by states at the Fermi energy;
4. The conductance does not change much with Fermi energy (plateau);

}

Then {

Changes in conductance (ΔG) ∝ local density of states (LDOS).

$$\Delta G\left(\boldsymbol{r_0}\right) \approx V_0 \frac{\partial G}{\partial E_F} \left|\varphi_n\left(\boldsymbol{r_n}\right)\right|^2$$



}

But…

Clouds are not spheres, mountains are not cones, coastlines are not circles, barks are not smooth, lighting does not travel in a straight line,…

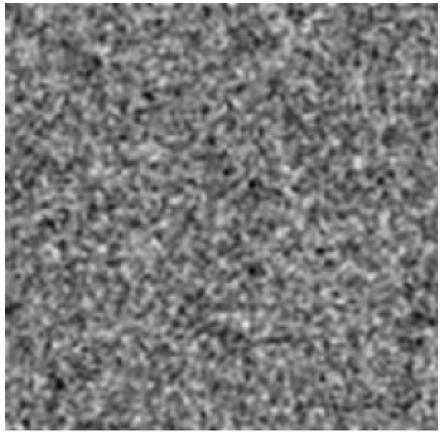
B. Mandelbrot
(1924 – 2010)

… ΔG is not small, the tip potential is not delta-shaped, not all states are in the Fermi level, and the conductance changes considerably with the Fermi level.

Machine Learning

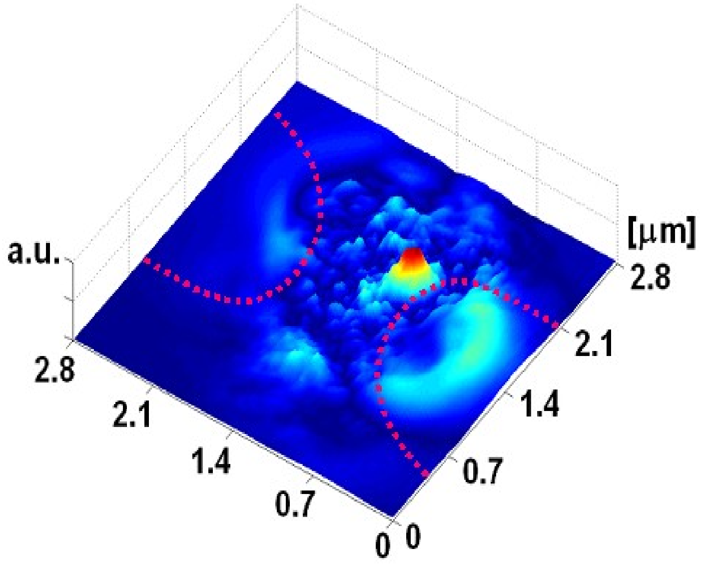
- Cellular Neural Networks
- Stigmergy Meta-heuristics

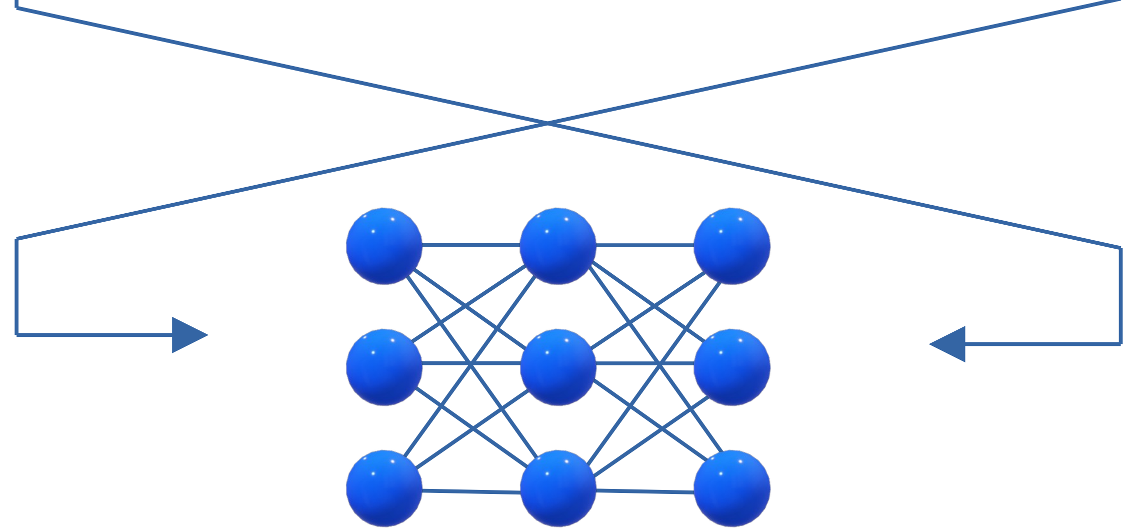
Inverse Problem



Random Potentials

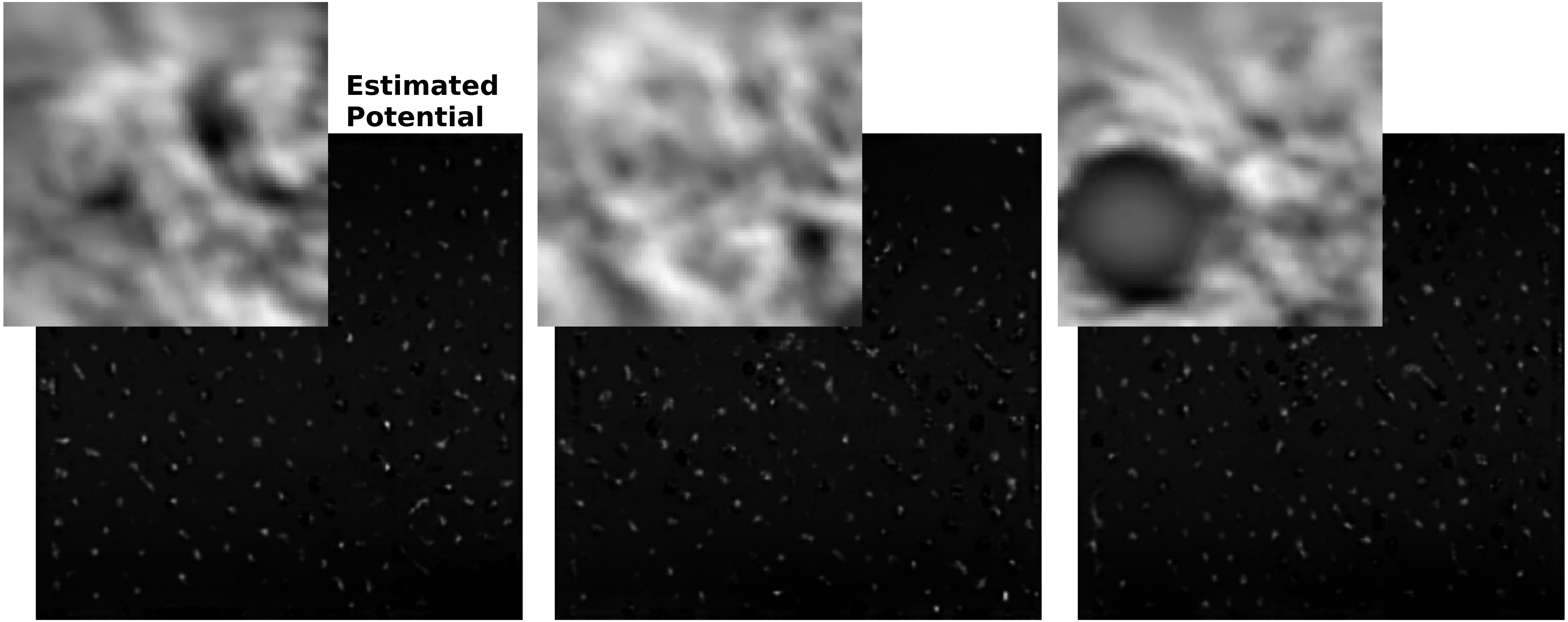
$$G^0_{nn}(E)=\left[(E+i\eta)I-H_n\right]^{-1}$$





Nonlinear array of elements

SGM

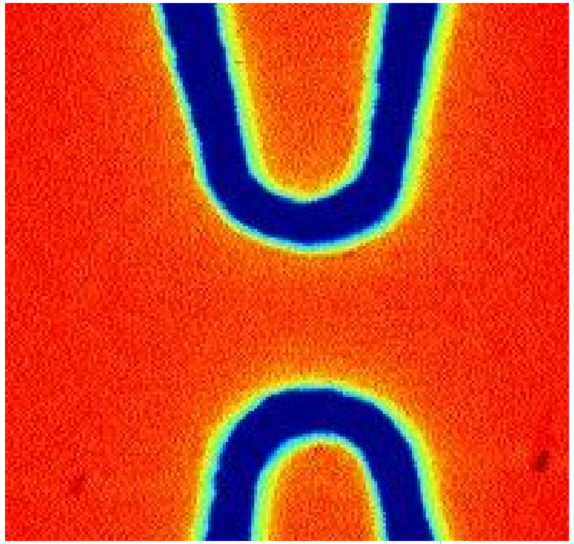


Estimated Potential

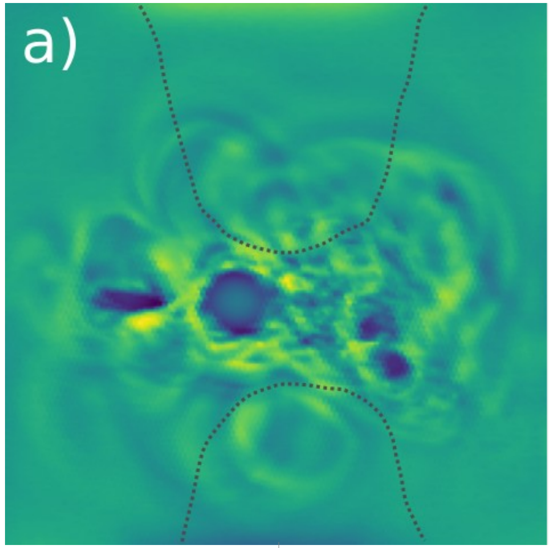
Slow processing limits the number of training samples!

Cellular Neural Network

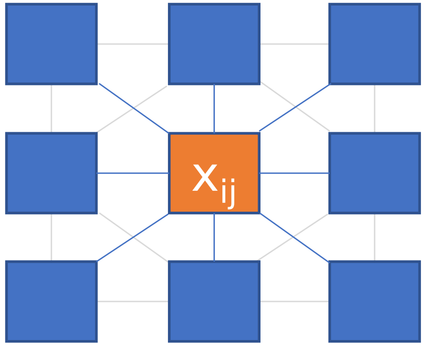
Theoretical Potential



Theoretical LDOS

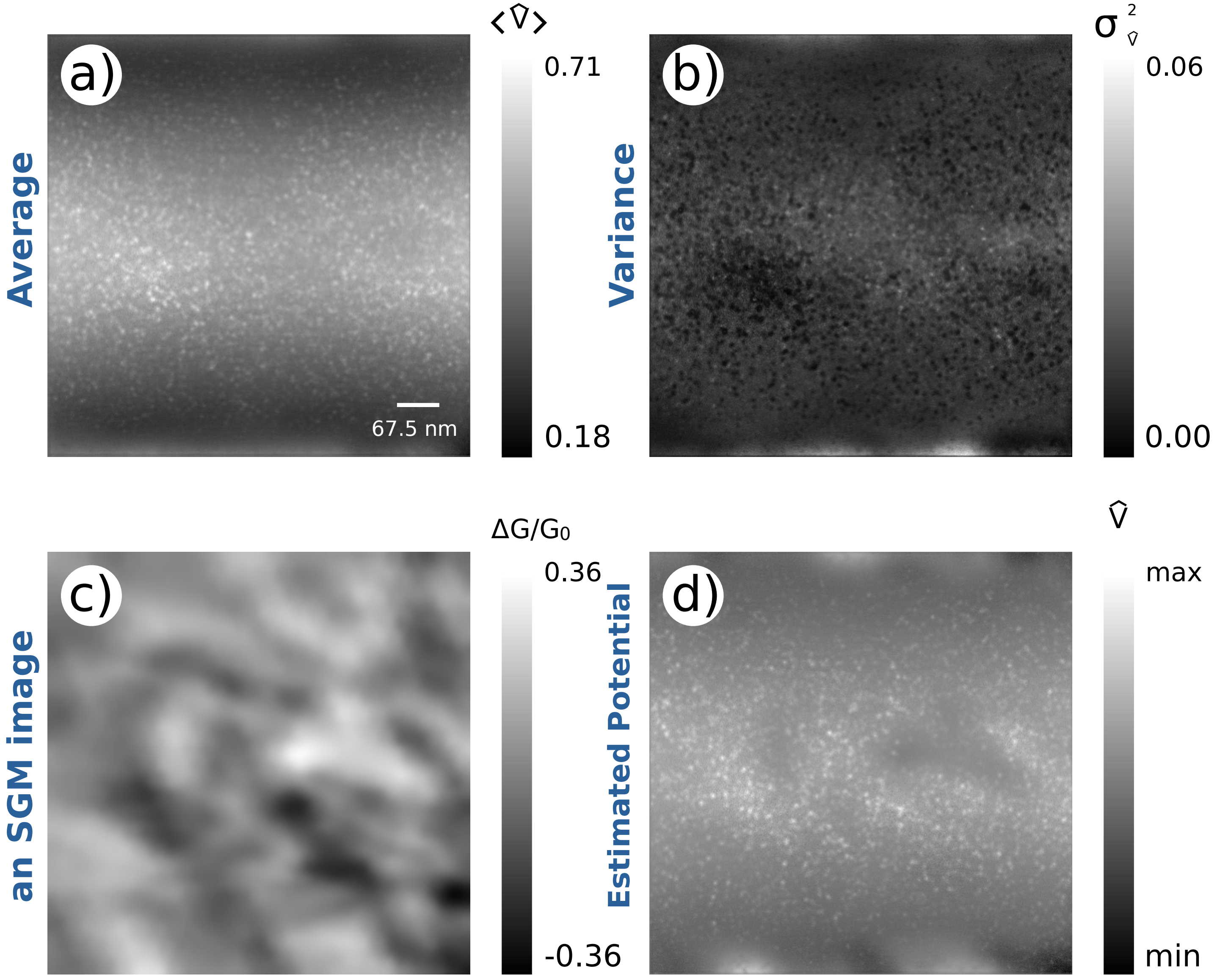


10k pairs with random Gaussian noise.



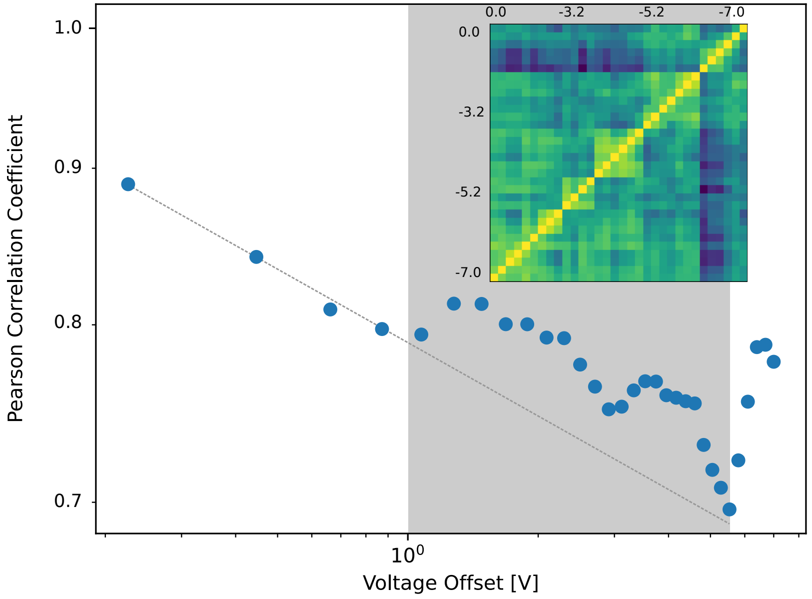
$$\frac{dX_{i,j}(t)}{dt} = -\underbrace{X_{i,j}(t)}_{\text{state}} + \boxed{\sum_{k,l} A_{i,j,k,l}\underbrace{Y_{k,l}(t)}_{\text{output}}} + \sum_{k,l} B_{i,j,k,l}\underbrace{U_{k,l}(t)}_{\text{input}} + Z$$

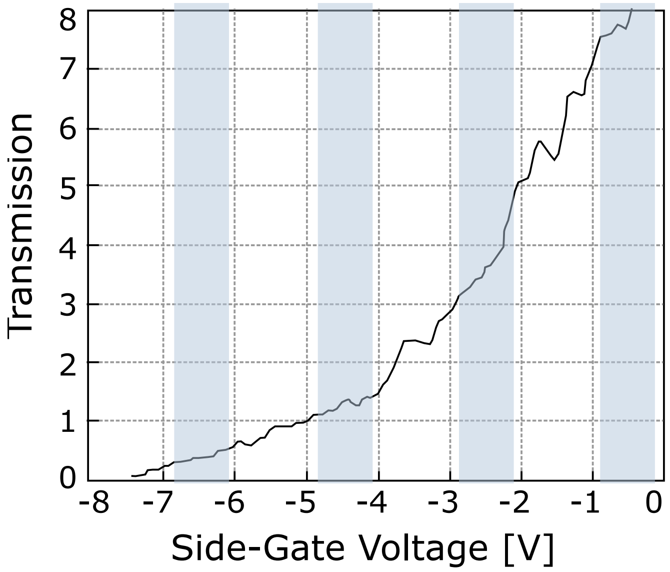
CNN ≡ Dynamical System. Attractor: X(t→∞)

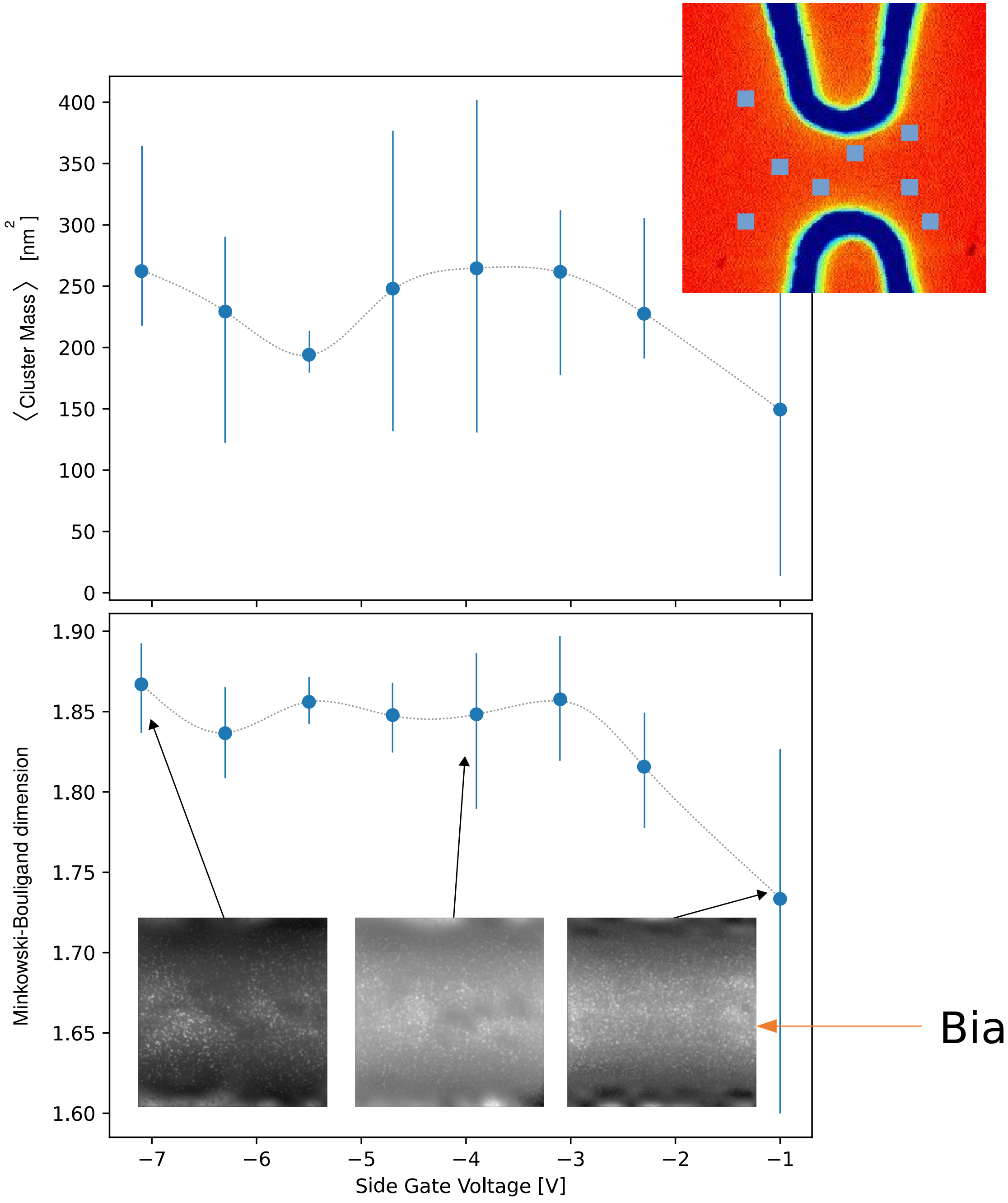


C. R. da Cunha, et al. A method for finding the background potential of quantum devices from scanning gate microscopy data using machine learning, *Mach. Learn. Sci. Tech.* **3** (2022) 025013.

Statistics

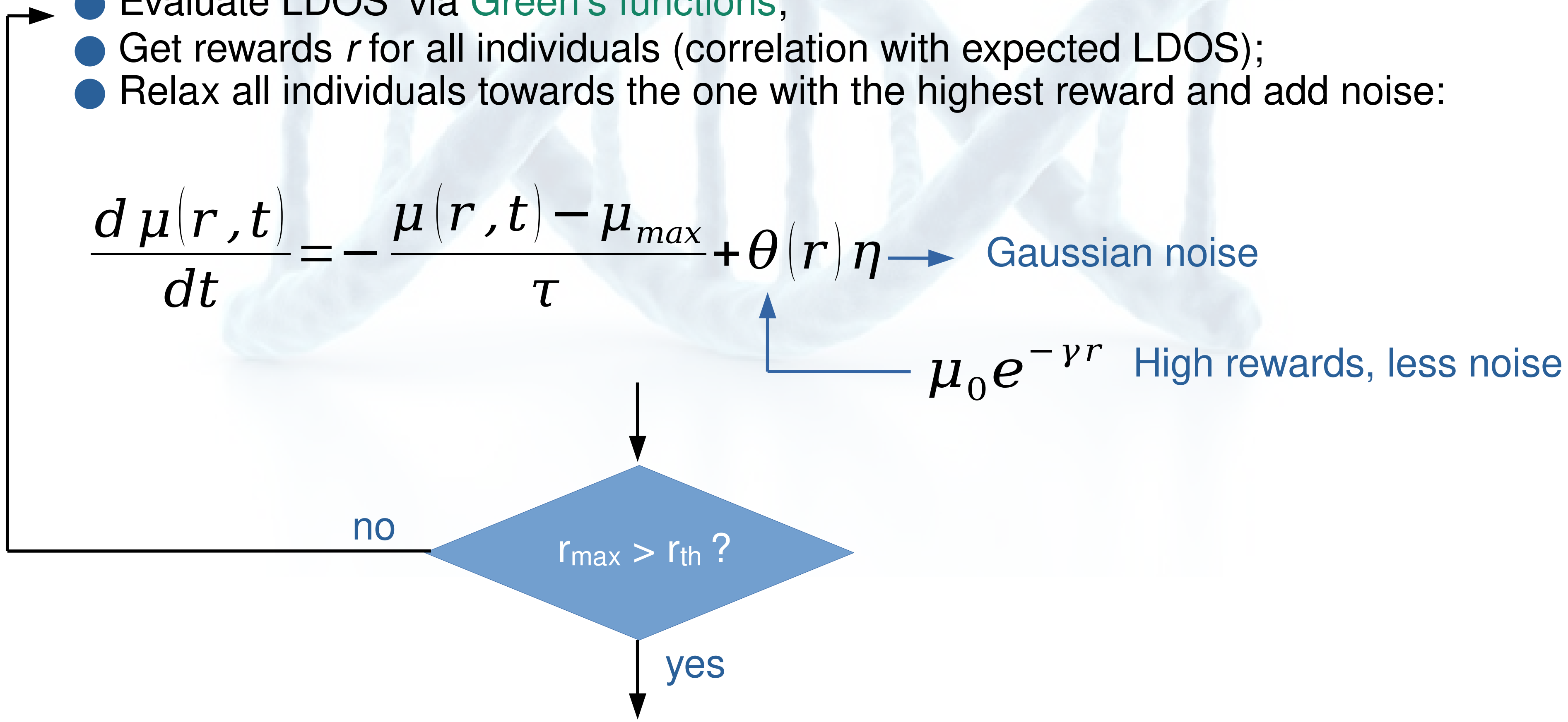






Biased!

Stigmergy Meta-Heuristics



- Generate random population (potentials μ);
- Evaluate LDOS’ via Green’s functions;
- Get rewards *r* for all individuals (correlation with expected LDOS);
- Relax all individuals towards the one with the highest reward and add noise:

$$\frac{d\mu(r,t)}{dt} = -\frac{\mu(r,t)-\mu_{max}}{\tau} + \theta(r)\eta$$

Gaussian noise

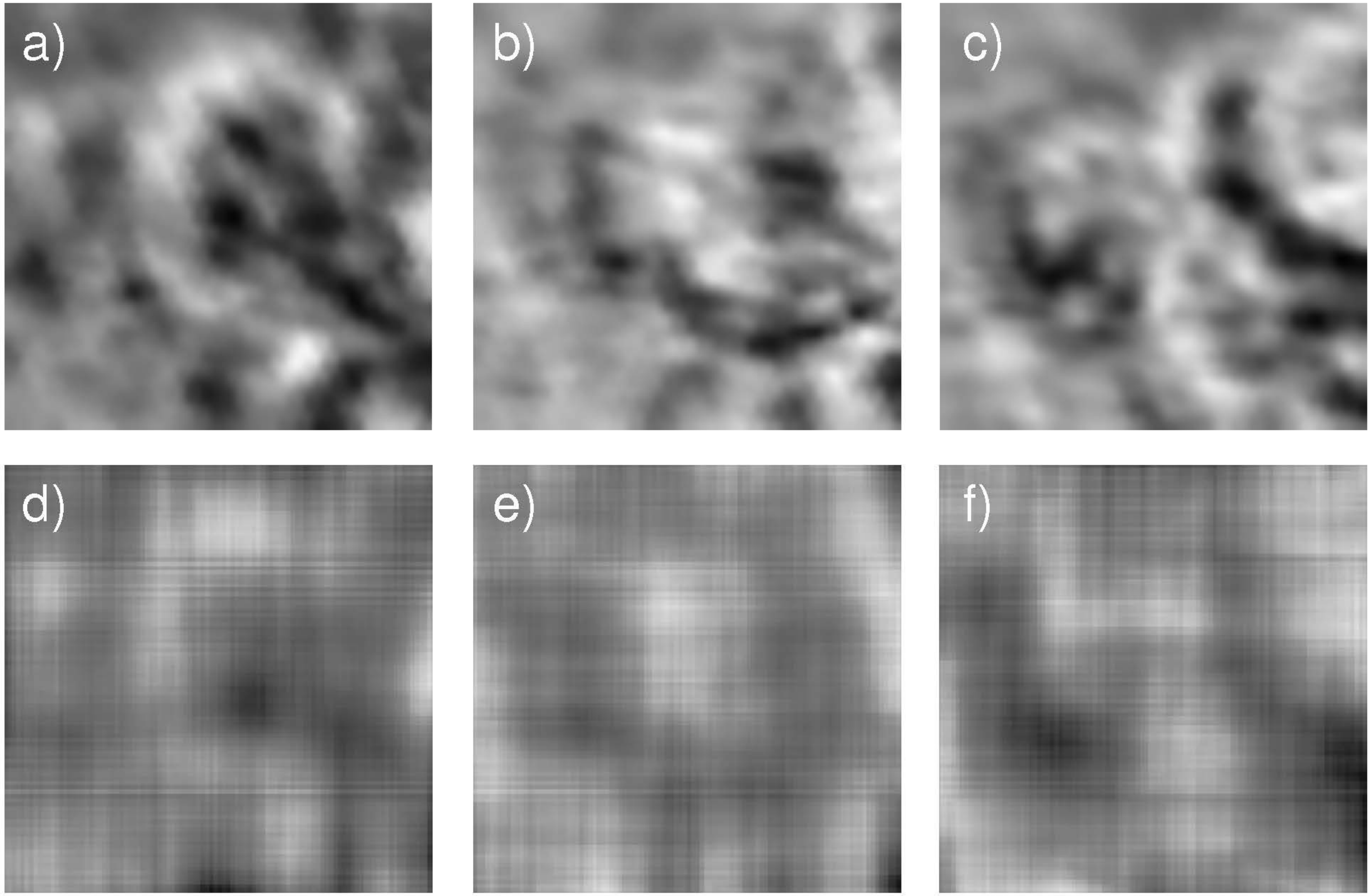
$\mu_0 e^{-\gamma r}$ High rewards, less noise

$r_{max} > r_{th}$?

no

yes

72 % Correlated SGM

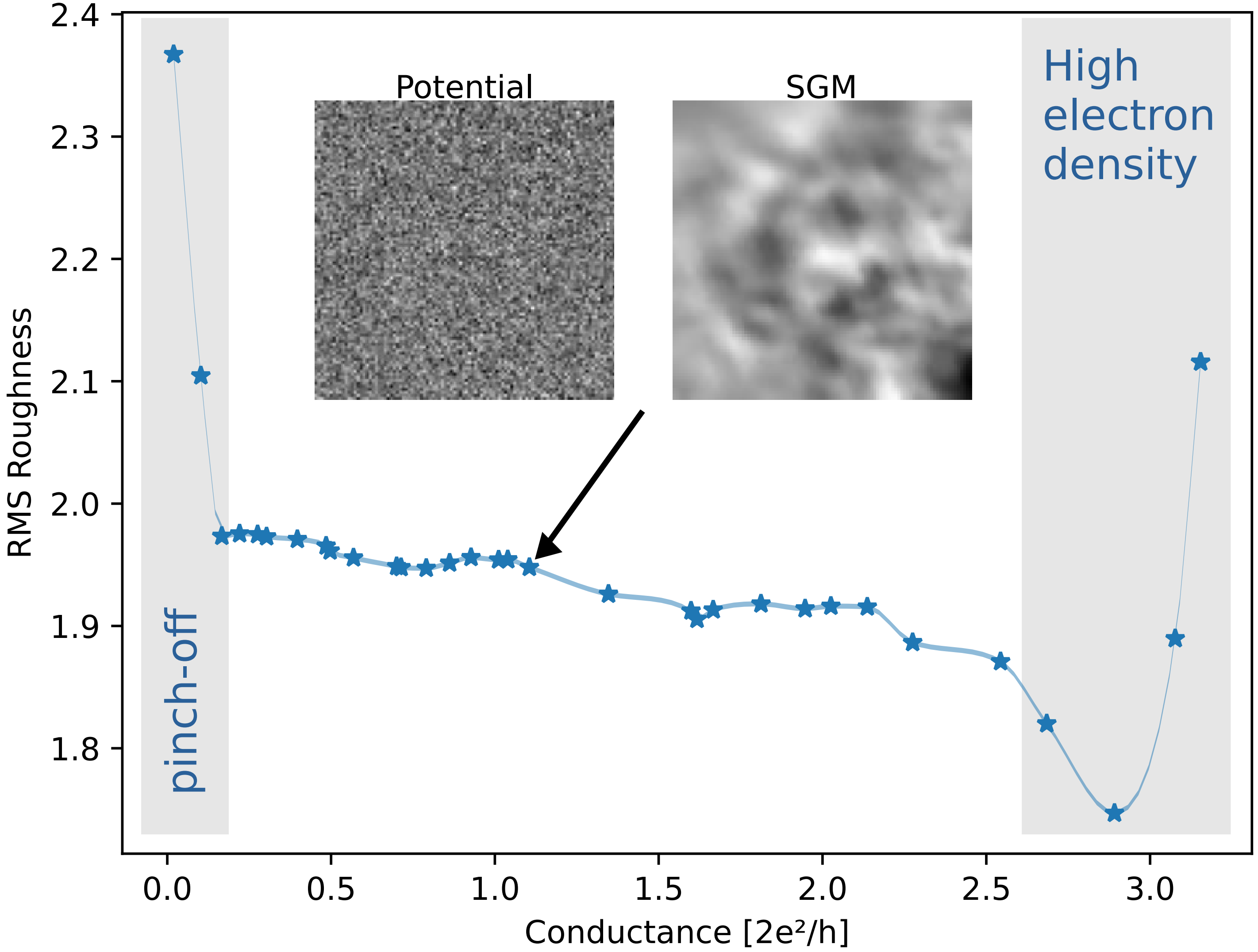


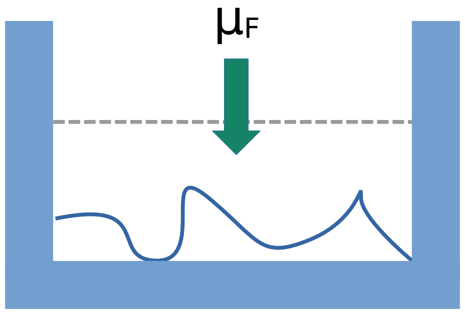
Experimental
(expected)

Generated
(winner)

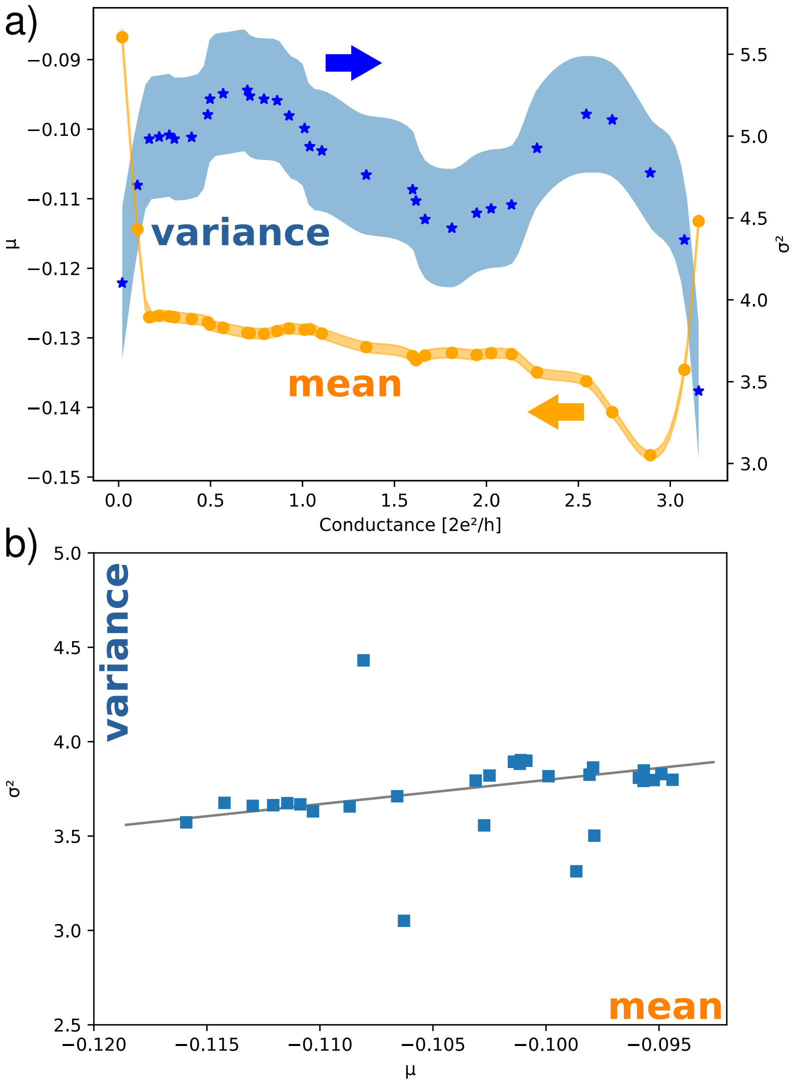
C. R. da Cunha, et al. An investigation of the background potential in quantum constrictions using scanning gate microscopy and a swarming algorithm, *Physica A* **614** (2023) 128550.

Potential Roughness





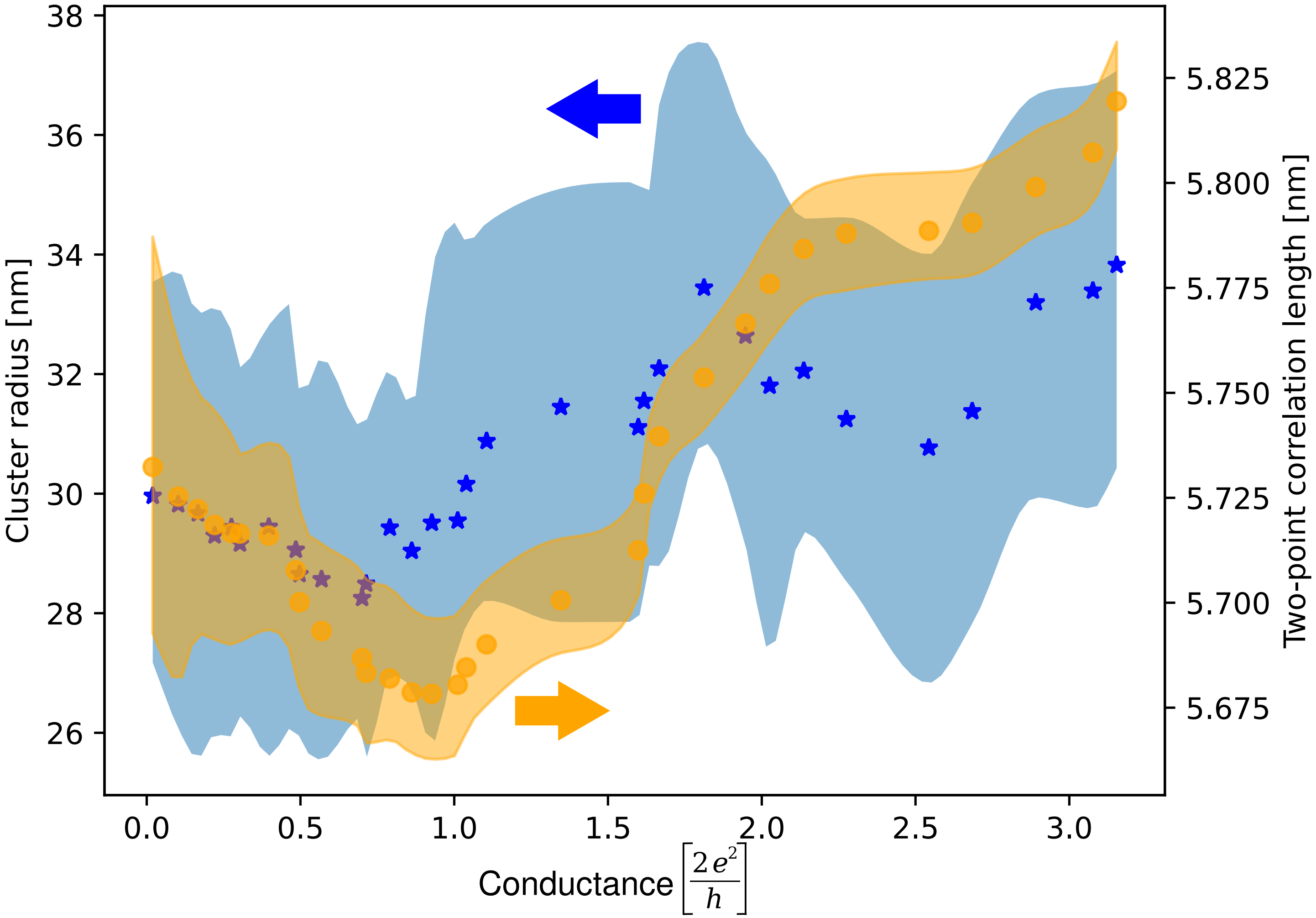
Potential Distribution



No fluctuation scaling.
Equally important points.

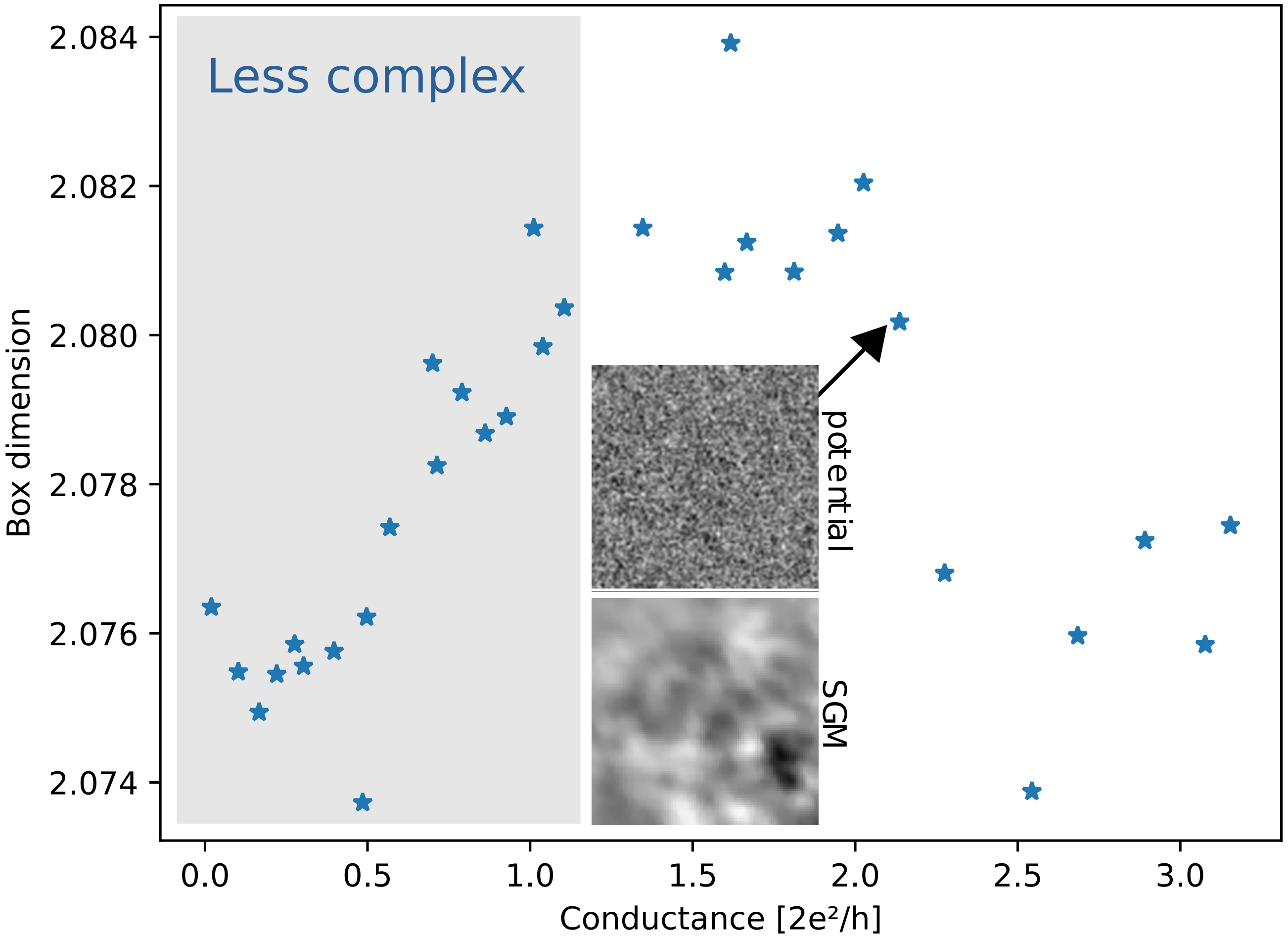
Two-Point Correlation Function

~ Bohr radius



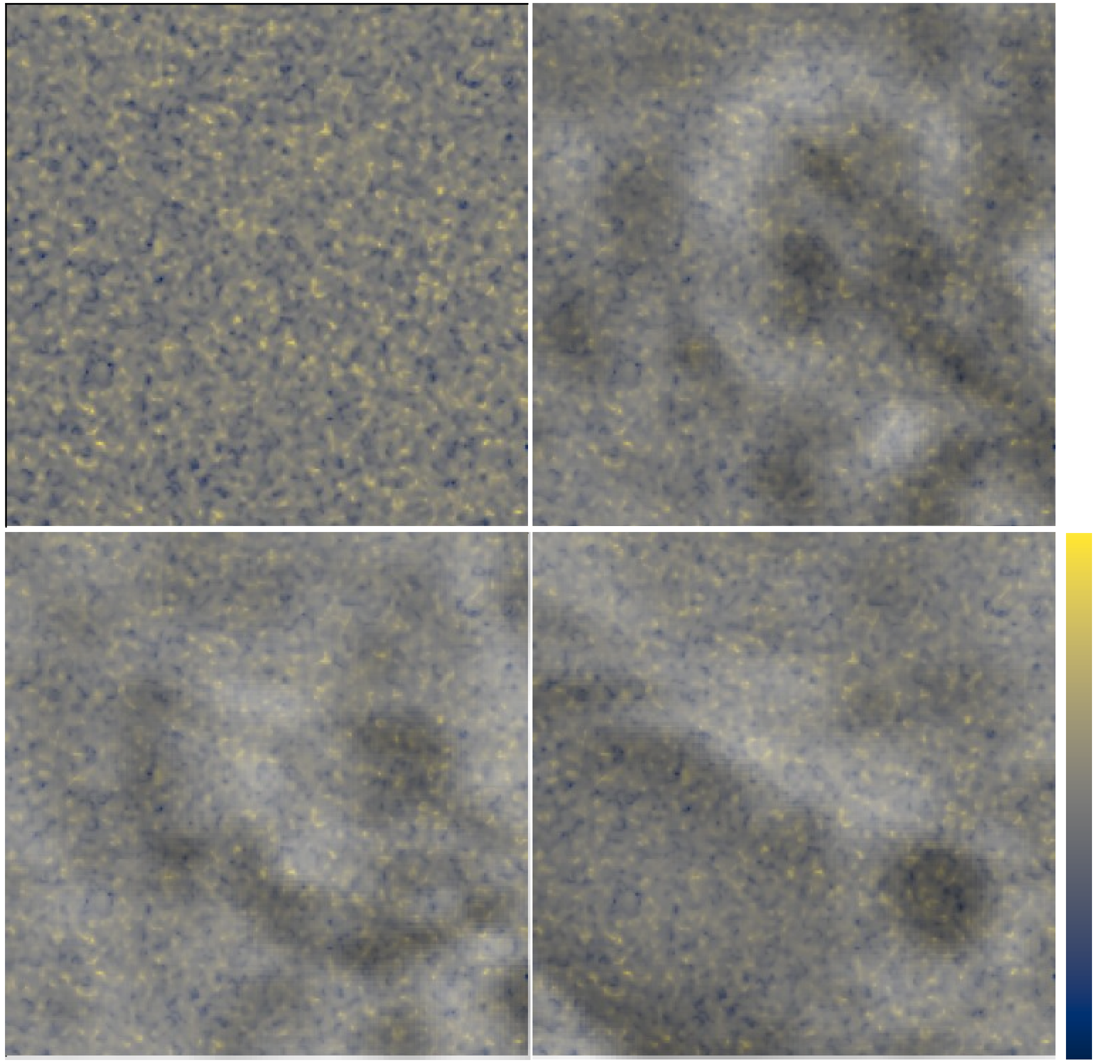
“clumpiness”

Box Dimension



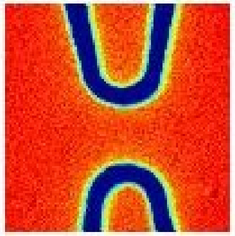
Static Alloy Potential

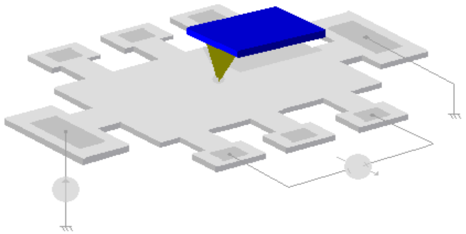
$$U_{ij}^{*} = \frac{\langle U_{ij} \rangle}{\sigma_{ij}}$$

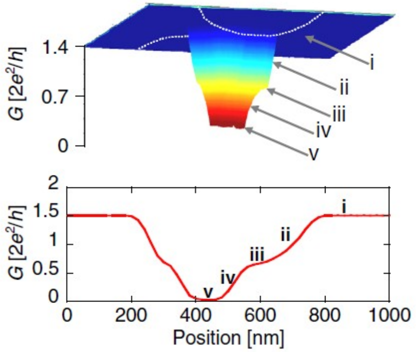


SGM images seem to be influenced by modes supported by the potential.

Conclusions







Quantum Point Contacts

- Case study: Disorder QPC
- SGM: Not simple interpretation
- Standard convolution layers not adequate
- Cellular neural networks are more adequate
- Swarming algorithm gets closer to reality (> 72 %)
- Rough potentials influence images at small base conductance
- All points in the disordered potential are equally important
- SGM images seem to be influenced by modes of the potential

Machine learning

Useful tool for inverse problems (if properly used)
Inverse design of new devices and materials!

Thank You

C·Lab

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“Nullius addictus iurare in verba magistri, quo me cumque rapit tempestas, deferor hospes.”
H. Flaccus