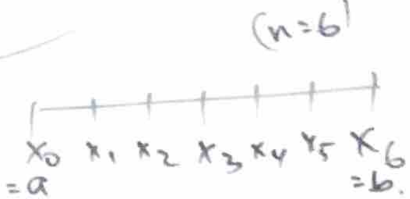
Limit Def of the Definite Integral.

Assume f is continuous on $[a,b]$

$$\int_a^b f(x)\,dx = \lim_{n\to\infty} M_n = \lim_{n\to\infty} L_n = \lim_{n\to\infty} R_n .$$

Note: This is actually a theorem. The Integral was defined last time



Given a, b, and the function f,
M_n, L_n and R_n are all Riemann sums.

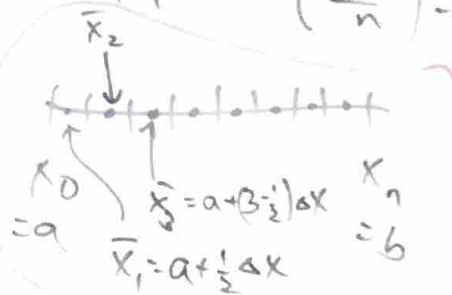
Choose n. Define $\Delta x = \dfrac{b-a}{n}$, $x_0 = a$, $X_1 = a + \Delta x$, $X_2 = a + 2\Delta x$, $X_i = a + i\Delta x$ $\therefore$ $X_n = b$

$\left(X_n = a + n\left(\dfrac{b-a}{n}\right) = a + b - a = b\right)$

$$L_n = \sum_{i=0}^{n-1} f(x_i)\,\Delta x, \qquad R_n = \sum_{i=1}^{n} f(x_i)\,\Delta x$$

Let $\bar{x}_i = x_i - \dfrac{\Delta x}{2} = a + \left(i - \tfrac{1}{2}\right)\Delta x$ $\qquad M_n = \displaystyle\sum_{i=1}^{n} f(\bar{x}_i)\,\Delta x$

Note that $\bar{x}_i$ is the midpoint of the i^{th} interval.



Note: L_n, R_n and M_n each have n terms in the sum.