MAT 136 (Calculus I), Prof. Jim Swift
Worksheet 9: Limits at infinity

Purple is required
Green is optional

Evaluate the limit. For full credit (on an exam or quiz), use the rule:

If $\lim_{x\to\infty} f(x)$ and $\lim_{x\to\infty} g(x)$ both exist, and $\lim_{x\to\infty} g(x) \neq 0$, then $\lim_{x\to\infty} \frac{f(x)}{g(x)} = \frac{\lim_{x\to\infty} f(x)}{\lim_{x\to\infty} g(x)}$.

$$\lim_{x\to\infty} \frac{2x^2+3x-1}{3x^2-x-4} = \lim_{x\to\infty} \frac{x^2(2+\frac{3}{x}-\frac{1}{x^2})}{x^2(3-\frac{1}{x}-\frac{4}{x^2})}$$

same as original for $x>0$

$$= \lim_{x\to\infty} \frac{(2+\frac{3}{x}-\frac{1}{x^2})}{(3-\frac{1}{x}-\frac{4}{x^2})}$$

cancelling x^2 from top & bottom

$$= \frac{\lim_{x\to\infty}(2+\frac{3}{x}-\frac{1}{x^2})}{\lim_{x\to\infty}(3-\frac{1}{x}-\frac{4}{x^2})}$$

This uses the rule mentioned above

$$= \boxed{\frac{2}{3}}$$

This is enough work, but see below for more details, using limit laws.

$$\lim_{x\to\infty}\left(2+\frac{3}{x}-\frac{1}{x^2}\right) = \lim_{x\to\infty}(2) + \lim_{x\to\infty}\left(\frac{3}{x}\right) - \lim_{x\to\infty}\left(\frac{1}{x^2}\right)$$

$$= \lim_{x\to\infty}(2) + 3\lim_{x\to\infty}\left(\frac{1}{x}\right) - \lim_{x\to\infty}\left(\frac{1}{x^2}\right)$$

$$= 2 + 3\cdot 0 - 0$$

$$= 2$$

Note: If $a>0$, then $\lim_{x\to\infty} x^a = \infty$ and $\lim_{x\to\infty} \frac{1}{x^a} = 0$