MAT 136 (Calculus I), Prof. Jim Swift
In-Class Worksheet: Derivative Shortcuts 5.

1. The function $y = \sin(x^2)$ is a composition of functions, $y = f(g(x))$, with

$f(u) =$ sin(u) and $g(x) =$ x^2 . Compute the derivatives of f and g:

$f'(u) =$ cos(u) and $g'(x) =$ $2x$. Practice the "eff of ex" notation:

$f'(x) =$ ~~sin(x)~~ cos(x) , $f'(y) =$ cos(y) , $f'(3u) =$ cos(3u) , $f'(x^2) =$ cos(x^2)

Now evaluate the derivative, using the chain rule: $\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

$\frac{d}{dx} \sin(x^2) =$ $\cos(x^2) \cdot 2x$

To show work, write this as the first step.

$\frac{d}{dx} \sin(x^2) = \cos(x^2) \cdot \frac{d}{dx}[x^2]$.

$(a+b)^2 = a^2 + 2ab + b^2$

2. Let $h(x) = (x^2+3)^2$. Compute $h'(x)$ in two ways:

(a) By expanding $h(x)$ to write it as a polynomial in standard form and then differentiating with the "old" rules.

$h(x) = x^4 + 6x^2 + 9$

$h'(x) = 4x^3 + 12x$

(b) Using the chain rule. $h(x) = f(g(x))$, where $f(u) = u^2$, $g(x) = x^2+3$

$f'(u) = 2u$, $g'(x) = 2x$

$f'(g(x)) = 2(x^2+3)$

$h'(x) = 2(x^2+3) \cdot 2x$

$= 4x(x^2+3)$

3. Differentiate $h(x) = (x^2+3)^{10}$. Note: One of methods (a) or (b) is very very much work.

$h(x) = x^{20} + \dots + 3^{10}$ is a complicated mess! Use method (b)!

$h'(x) = 10(x^2+3)^9 \cdot 2x$

since $f(u) = u^{10}$, $g(x) = x^2+3$

$f'(u) = 10u^9$ $g'(x) = 2x$

$f'(x^2+3) = 10(x^2+3)^9$.