MAT 136 (Calculus I), Prof. Jim Swift, In-Class Worksheet: Derivative of inverse trig functions, and the tangent line to a curve

1. Find the derivative of these functions:

$f(x) = x \arctan(x^2)$. $f'(x) = 1 \cdot \arctan(x^2) + x \cdot \frac{1}{1+(x^2)^2} \cdot \frac{d}{dx}(x^2)$

$= \boxed{\arctan(x^2) + \frac{2x^2}{1+x^4}}$

$g(x) = \ln|\arcsin(2x)|$

$g'(x) = \frac{1}{\arcsin(2x)} \cdot \frac{d}{dx}\arcsin(2x) = \frac{1}{\arcsin(2x)} \cdot \frac{1}{\sqrt{1-(2x)^2}} \cdot \frac{d}{dx}(2x) = \boxed{\frac{2}{\arcsin(2x)\sqrt{1-4x^2}}}$

2. Find an equation of the tangent line to the curve $x^2 + xy^2 + y^3 = 7$ at the point $(x, y) = (2, 1)$.

$(x_0, y_0) = (2,1)$

$$\boxed{y = m(x-2)+1 \text{ where } m = \left.\frac{dy}{dx}\right|_{\substack{x=2\\y=1}}}$$

The low-hanging fruit!

$$\frac{d}{dx}(x^2+xy^2+y^3) = \frac{d}{dx}[7]$$

$$2x + \left(1\cdot y^2 + x \cdot 2y\frac{dy}{dx}\right) + 3y^2\frac{dy}{dx} = 0$$

solve for $\frac{dy}{dx}$. It's linear!

$$2x + y^2 + (2xy + 3y^2)\frac{dy}{dx} = 0$$

so $$\frac{dy}{dx} = \frac{-(2x+y^2)}{2xy+3y^2}$$

$$m = \left.\frac{dy}{dx}\right|_{\substack{x=2\\y=1}} = \frac{-(2\cdot 2+1^2)}{2\cdot 2\cdot 1 + 3\cdot 1^2} = \frac{-5}{7}$$

So, an equation of the tangent line is $\boxed{y = \frac{-5}{7}(x-2)+1}$

See Desmos graph.