MAT 136 (Calculus I) Prof. Swift

In-class worksheet on Local Linearization: Feynman vs. the Abacus

First, read the story of Feynman vs. the Abacus. (Available on our web site, or via google search.) This worksheet will approximate $\sqrt[3]{1729.03}$ using a method equivalent to what Feynman did, but following the notation we use in our class. Use the facts, known by Feynman, that $12^2 = 144$ and $12^3 = 1728$.

1. (no technology) We want to compute $\sqrt[3]{1729.03}$. Define the function $f(x) = \sqrt[3]{x}$, so we want to approximate $f(1729.03)$. Feynman knew that

$f(1728) = \sqrt[3]{1728} = \underline{12}$. (since $12^3 = 1728$)

This is the first approximation for $\sqrt[3]{1729.03}$, and getting this approximation brought sweat to the forehead of the guy with the abacus.

2. (no technology) To get a better approximation, we will use the local linearization $f(x) \approx \ell_{1728}(x) = f(1728) + f'(1728) \cdot (x - 1728)$. We already know $f(1728)$, so we just have to find $f'(1728)$. Note that $f(x) = x^{1/3}$, and compute $f'(x)$ below.

$f'(x) = \frac{1}{3} x^{-\frac{2}{3}}$ (since $\frac{1}{3} - 1 = -\frac{2}{3}$)

3. (no technology) Next, evaluate $f'(1728)$ without a calculator. Write your answer as a fraction of integers. No exponents are allowed! Hint: Remember that $(x^2)^3 = x^{2 \cdot 3}$.

$f'(1728) = \frac{1}{3}(1728)^{-\frac{2}{3}} = \frac{1}{3}\left((1728)^{\frac{1}{3}}\right)^{-2} = \frac{1}{3} \cdot 12^{-2} = \frac{1}{3 \cdot 12^2} = \boxed{\frac{1}{3 \cdot 144}} = \boxed{\frac{1}{432}}$

either one OK

4. (no technology) Putting the results of problems 1 and 3 together, the local linearization of $f(x)$ at $x = 1728$ is

$\ell_{1728}(x) = 12 + \frac{1}{3 \cdot 144}(x - 1728) = 12 + \frac{1}{432}(x - 1728)$ either one OK.

5. (no technology) Now, approximate the cube root of 1729.03. Evaluate $1729.03 - 1728 = 1.03$ in your head, and get an exact expression for $\ell_{1728}(1729.03)$ involving a fraction with 1.03 in the numerator.

$\sqrt[3]{1729.03} \approx \ell_{1728}(1729.03) = 12 + \frac{1}{3 \cdot 144}(1.03) = 12 + \frac{1.03}{3 \cdot 144} = 12 + \frac{1.03}{432}$

6. (yes technology) Now, unlike Feynman, use technologogy evaluate $\ell_{1728}(1729.03)$. Round to 9 significant figures. Use "..." for rounding, as in $\sqrt{2} = 1.41421356\ldots$. Feynman, without a calculator, computed the *correction*, Δf, to 1 significant figure.

$\sqrt[3]{1729.03} \approx 12.0023843\ldots$

7. (yes technology) Finally, find the true value of the cube root, rounded to 9 significant figures. (If you round to fewer than 9 significant figures, you get the same answer for 6 and 7.)

$\sqrt[3]{1729.03} = 12.0023838\ldots$

(Both 6. and 7. round to 12.002384..., to 8 significant figures.)