MAT 136 (Calculus I) Prof. Swift

In-class worksheet: Critical points and Local Extrema

Consider the function $f(x) = \frac{1}{3}x^3 + x^2 - 3x + 4$.

1. Complete the sentence: $f'(x) =$ x^2+2x-3

2. Find all of the critical points of f.

Solve $x^2+2x-3=0$

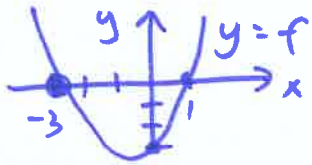
$(x+3)(x-1)=0$

$x=-3$ or $x=1$

The critical points are -3 and 1

3. Sketch $y = f'(x)$. Focus on the x-intercepts of this graph, and the sign of $f'(x)$.

Note: $f'(-3)=0$ and $f(1)=0$ and $f(0)=-3$.



4. Apply the first derivative test to each critical point of f.

$f'(-3)=0$ and f' is decreasing on $[-4,-2]$. therefore f has a local max at -3.

$f'(1)=0$ and f' is increasing on $[0,2]$. therefore f has a local min at 1.

5. Find $f''(x)$, and apply the second derivative test to each critical point of f.

$f''(x) = 2x+2$.

$f''(-3) = 2(-3)+2 = -6+2 = -4 < 0$ $\therefore$ f has a local max at -3.

$f''(1) = 2\cdot 1+2 = 4 > 0$ $\therefore$ f has a local min at 1.