MAT 136 (Calculus I) Prof. Swift

In-class worksheet: The Indefinite Integral, Part 2

1. Evaluate $\frac{d}{dt}\cos(t)$ $= -\sin(t)$

2. Evaluate $\int \sin(t)\,dt$ $= -\cos(t) + C$ Check: $\frac{d}{dt}(-\cos(t)) = -(-\sin(t)) = \sin(t)$ ✓

3. Evaluate $\frac{d}{dx}e^{3x}$ $= e^{3x} \cdot 3$ (from chain rule)

4. Evaluate $\int e^{3x}\,dx$ $= \frac{1}{3}e^{3x} + C$ (Check: $\frac{d}{dx}\frac{1}{3}e^{3x} = \frac{1}{3}e^{3x} \cdot 3 = e^{3x}$ ✓)

5. Consider the function $f(x) = \frac{1}{x^2} - \frac{1}{x^3}$ $= x^{-2} - x^{-3}$. Let $F(x)$ be the antiderivative of $f(x)$ with $F(1) = 0$. Find $F(x)$.

$F(x) = \frac{x^{-1}}{-1} - \frac{x^{-2}}{-2} + C = \frac{-1}{x} + \frac{1}{2x^2} + C$

$F(1) = \frac{-1}{1} + \frac{1}{2 \cdot 1^2} + C \overset{\text{set}}{=} 0$

$-1 + \frac{1}{2} + C = 0$

$C = 1 - \frac{1}{2} = \frac{1}{2}$

6. Solve the Initial Value Problem $\frac{dy}{dx} = \frac{1}{x^2} - \frac{1}{x^3}$, $y(1) = 0$.

5. $\boxed{F(x) = \frac{-1}{x} + \frac{1}{2x^2} + \frac{1}{2}}$

6. Same question! write down the solution immediately:

$\boxed{y = \frac{-1}{x} + \frac{1}{2x^2} + \frac{1}{2}}$, or $\boxed{y(x) = \frac{-1}{x} + \frac{1}{2x^2} + \frac{1}{2}}$

either solution is OK