MAT 136 (Calculus I) Prof. Swift

In-class worksheet: The Fundamental Theorem of Calculus, Part II

1. Find a simple formula for $f(x) = \int_1^x t^3\,dt$ $= \frac{t^4}{4}\Big|_1^x = \frac{x^4}{4} - \frac{1^4}{4} = \boxed{\frac{x^4}{4} - \frac{1}{4}}$

2. Evaluate $f(1)$ and $f'(x)$ for the function f you found in problem 1.

$f(1) = \frac{1}{4} - \frac{1}{4} = \boxed{0}$ $\quad f'(x) = \frac{4x^3}{4} - 0 = \boxed{x^3}$. ← This agrees with the FTOC II.

3. Let $g(x) = \int_2^x \sin(t^2)\,dt$. Evaluate $g(2)$ and $g'(x)$.

Hint: Do not attempt to find a simple formula for $g(x)$, like you did in problem 1.

$g(2) = \int_2^2 \sin(t^2)\,dt = \boxed{0}$, $\boxed{g'(x) = \sin(x^2)}$ from the FTOC II.

4. Let $h(x) = \int_2^{x^2} \sin(t^2)\,dt$. Evaluate $h(\sqrt{2})$ and $h'(x)$.

Hint: $h(x) = g(x^2)$. $h(\sqrt{2}) = g((\sqrt{2})^2) = g(2) = 0$.

$h'(x) = g'(x^2) \cdot 2x$ (from the chain rule.)

$= \sin((x^2)^2) \cdot 2x$ (from the formula for $g'(x)$.)

$\boxed{h'(x) = 2x \sin(x^4)}$