MAT 136 (Calculus I) Prof. Swift

In-class worksheet: The method of u-substitution

1. Evaluate the indefinite integral using the substitution $u = x^2 + 3$.

$$\int x \sec^2(x^2+3)\,dx = \frac{1}{2}\int \sec^2(u)\,du = \frac{1}{2}\tan(u) + C = \boxed{\frac{1}{2}\tan(x^2+3)+C}$$

$$= \boxed{\frac{1}{2}\tan(x^2+3)+C}$$

$u = x^2+3$

$du = 2x\,dx$, so $x\,dx = \frac{1}{2}du$

2. Evaluate the indefinite integral using the substitution $u = \cos(x)$.

$$\int \tan(x)\,dx = \int \frac{\sin(x)}{\cos(x)}\,dx = \int \frac{1}{u}(-du) = -\int \frac{1}{u}du = -\ln|u| + C$$

$$= \boxed{-\ln|\cos(x)| + C}$$

$u = \cos(x)$

$du = -\sin(x)dx$

3. Evaluate the indefinite integral using the substitution $u = e^{2x}$.

$$\int \frac{e^{2x}}{1+e^{4x}}\,dx = \int \frac{\frac{1}{2}du}{1+u^2} = \frac{1}{2}\int \frac{1}{1+u^2}du = \frac{1}{2}\arctan(u) + C$$

$$= \boxed{\frac{1}{2}\arctan(e^{2x}) + C}$$

$u = e^{2x}$, $e^{4x} = (e^{2x})^2 = u^2$

$du = 2e^{2x}dx$

$e^{2x}dx = \frac{1}{2}du$