

This is Swift's solutions, using rules as he taught them.

ID Check _____

Time Started _____

Name _____

Section/Professor _____

MAT 136: CALCULUS I
GATEWAY EXAM - VERSION D

Full Pass requires 7 or 8 problems; Half Pass 5 or 6. No partial credit

NO CALCULATOR OR SCRATCH PAPER

BOX FINAL ANSWERS

In each case, find the derivative of the given function. It is not necessary to simplify the result algebraically.

1. $f(x) = \frac{x^2 - 3x + 2}{x^2 + 2}$; $f'(x) = \frac{\frac{d}{dx}[x^2 - 3x + 2](x^2 + 2) - (x^2 - 3x + 2)\frac{d}{dx}[x^2 + 2]}{(x^2 + 2)^2}$

This can be simplified, but don't simplify here →

$$= \frac{(2x - 3)(x^2 + 2) - (x^2 - 3x + 2)(2x)}{(x^2 + 2)^2}$$

2. $f(x) = e^{\cos(x)} \cos(x)$

$$\begin{aligned} f'(x) &= \frac{d}{dx}[e^{\cos(x)}] \cos(x) + e^{\cos(x)} \frac{d}{dx}[\cos(x)] \\ &= e^{\cos(x)} \frac{d}{dx}[\cos(x)] \cdot \cos(x) + e^{\cos(x)} (-\sin(x)) \\ &= \boxed{e^{\cos(x)} (-\sin(x)) \cdot \cos(x) + e^{\cos(x)} (-\sin(x))} = \boxed{e^{\cos(x)} \sin(x) (\cos(x) + 1)} \end{aligned}$$

OK OK too

3. $g(x) = \ln(\tan(x))$

$$\begin{aligned} g'(x) &= \frac{1}{\tan(x)} \cdot \frac{d}{dx} \tan(x) = \boxed{\frac{1}{\tan(x)} \cdot \frac{1}{\cos^2(x)}} \text{ OK} \\ &= \boxed{\frac{1}{\tan(x)} \cdot \sec^2(x)} \text{ OK} \\ &= \frac{\cos(x)}{\sin(x)} \cdot \frac{1}{\cos^2(x)} = \boxed{\frac{1}{\sin(x) \cos(x)}} \end{aligned}$$

4. $f(t) = \frac{t^{3/2} - t + t^{1/2}}{\sqrt{t^3}} = \frac{t^{3/2} - t + t^{1/2}}{t^{3/2}} = t^{-3/2} (t^{3/2} - t + t^{1/2}) = 1 - t^{-1/2} + t^{-1}$

so $f'(t) = \boxed{\frac{1}{2} t^{-3/2} - t^{-2}}$

Quotient rule can be used, but it is much more work.

Note: Memorize or be able to derive the formula $\frac{d}{dx} a^x = \ln(a) \cdot a^x$

Derivation:

$$\frac{d}{dx} a^x = \frac{d}{dx} [e^{\ln(a^x)}] = \frac{d}{dx} [e^{x \ln(a)}] = e^{x \ln(a)} \cdot \frac{d}{dx} [x \ln(a)] = e^{x \ln(a)} \cdot \ln(a) = a^x \cdot \ln(a)$$

5. $f(x) = \sin(\ln(x^2 + 1))$

$$f'(x) = \cos(\ln(x^2 + 1)) \cdot \frac{d}{dx} [\ln(x^2 + 1)]$$

$$= \cos(\ln(x^2 + 1)) \cdot \frac{1}{x^2 + 1} \frac{d}{dx} [x^2 + 1]$$

$$= \boxed{\cos(\ln(x^2 + 1)) \cdot \frac{1}{x^2 + 1} \cdot 2x} = \boxed{\cos(\ln(x^2 + 1)) \cdot \frac{2x}{x^2 + 1}}$$

6. $f(x) = \arctan(x^2)$

$$f'(x) = \frac{1}{1 + (x^2)^2} \cdot \frac{d}{dx} [x^2] = \boxed{\frac{1}{1 + x^4} \cdot 2x} = \boxed{\frac{2x}{1 + x^4}}$$

7. $g(t) = e^{e^{t^2}} = e^{\exp(t^2)}$, using $\exp(x) = e^x$ notation.

$$g'(t) = e^{e^{t^2}} \cdot \frac{d}{dt} [e^{t^2}] = e^{e^{t^2}} \cdot e^{t^2} \cdot \frac{d}{dt} [t^2] = \boxed{e^{e^{t^2}} \cdot e^{t^2} \cdot 2t}$$

8. $f(t) = 4^t \cos(4t)$ see note above. $\frac{d}{dt} 4^t = \ln(4) \cdot 4^t$.

$$f'(t) = \frac{d}{dt} [4^t] \cos(4t) + 4^t \frac{d}{dt} \cos(4t)$$

~~$$= \ln(4) \cdot 4^t \cos(4t) + 4^t (-\sin(4t)) \frac{d}{dt} (4t)$$~~

~~$$= \ln(4) \cdot 4^t \cos(4t) - 4^t \sin(4t) \cdot 4$$~~ OK

$$= \boxed{4^t (\ln(4) \cos(4t) - 4 \sin(4t))}$$
 OK TOO