MAT 136 (Calculus I), Prof. Jim Swift
Worksheet 3, Modeling US Population, and Inverse Functions

1. The population of the United States was approximately 148 million in 1950 and was 282 million in 2000. approximately 336 million in 2020. Open the Desmos Graphing calculator link that plots these two data points.

Let $y = f(t)$ be the population of the US, measured in millions, in the year t. Thus $f(1950) = 148$ and $f(2020) = 336$.

(a) Modify the first function to plot a linear model $y = f(t)$ that fits these two data points. Use the slope-intercept form $y = m(t - t_0) + y_0$.

(b) Modify the second function to plot an exponential model $y = f(t)$ that fits these two data points. Use the form $y = y_0 \cdot a^{(t-t_0)/h}$.

(c) Plot another data point showing the census data of 336 million in 2020. What do you notice?

(d) Plot another data point showing the census data of 76 million in 1900. What do you notice?

(e) Why didn't we use the models $y = mt + b$ and $y = a \cdot b^t$?

2. Let $f(x) = x^3 + 2x - 1$. Make a new file on Desmos and plot $y = f(x)$.

(a) Answer this next question by simply looking at the graph. Is f one-to-one?

Recall that if f is a one-to-one function, then the inverse function f^{-1} exists, and it is defined by $x = f^{-1}(y)$ if and only if $y = f(x)$.

Your answer to part (a) implies that f^{-1} exists. But we cannot solve $y = x^3 + x - 1$ for x with pencil and paper, so we cannot find a formula for $f^{-1}(y)$, or for $f^{-1}(x)$.

(Evidently, Girolamo Cardano (1501-1576) found a way to solve that equation, but the formula is too complicated to be of much use.)

(b) Fill in the blanks with exact answers. $f(2) =$ ____. This tells us that $f^{-1}(9) =$ ____.

(c) Use the Desmos graph to estimate $f^{-1}(2)$ to 4 significant figures: $f^{-1}(2) =$ _______.