MAT 136 (Calculus I), Prof. Jim Swift
Worksheet 7, on Continuity and Algebraic Limits.

1. The function $f(x) = e^{\cos(x)}$ is continuous on the set of all real numbers. Evaluate the limit.

$\lim_{x\to 1} e^{\cos(x)} =$ $e^{\cos(1)}$, since f is continuous at $x=1$

2. The function $f(x) = e^{-1/x^2}$ is continuous on its domain. Note that $f(0)$ is undefined. Can we conclude that $\lim_{x\to 0} f(x)$ DNE? NO. Why or why not? Because "$f(a)$ is undefined" is allowed when $\lim_{x\to a} f(x)$ exists.

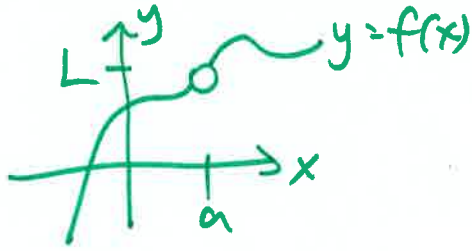
3. Consider the function f defined by $f(x) = \frac{x^2-4}{x-2}$.

(a) What is the default domain of f? $x \neq 2$. $(-\infty,2)\cup(2,\infty)$

(b) Find a function $\tilde{f}$ such that:

- The domain of $\tilde{f}$ is all real numbers, and
- $f(x) = \tilde{f}(x)$ for all x in the domain of f.

Here's a graph



$\lim_{x\to a} f(x) = L$, and $f(a)$ is undefined.

$f(x) = \frac{x^2-4}{x-2} = \frac{(x+2)(x-2)}{x-2}$

$= x+2$, provided $x \neq 2$

That is,

$$f(x) = \begin{cases} x+2 & \text{if } x \neq 2 \\ \text{undefined} & \text{if } x = 2 \end{cases}$$

So $\tilde{f}(x) = x+2$ satisfies both bulleted conditions.