Worksheet 14 Solutions

1. $f(x)=(x^2+3x+1)(x^3-2x+1)$. You can expand out, but product rule is easier

$$f'(x)=\frac{d}{dx}[x^2+3x+1](x^2-2x+1)+(x^2+3x+1)\frac{d}{dx}[x^3-2x+1]$$

$$\boxed{f'(x)=(2x+3)(x^2-2x+1)+(x^2+3x+1)(3x^2-2)}$$

2. $y=x(e^x-x^2)$

$$\frac{dy}{dx}=\frac{d}{dx}[x](e^x-x^2)+x\frac{d}{dx}[e^x-x^2]$$

$$\boxed{\frac{dy}{dx}=1(e^x-x^2)+x(e^x-2x)}$$

3. $h(x)=\dfrac{x}{x^2+2}$

$$h'(x)=\frac{\frac{d}{dx}[x](x^2+2)-x\frac{d}{dx}[x^2+2]}{(x^2+2)^2}=\frac{(x^2+2)-x(2x)}{(x^2+2)^2}$$

$$=\frac{x^2+2-2x^2}{(x^2+2)^2}=\boxed{\frac{-x^2+2}{(x^2+2)^2}}$$

4. $f(x)=e^{2x}=e^x\cdot e^x$; $f'(x)=\frac{d}{dx}[e^x]\cdot e^x+e^x\cdot\frac{d}{dx}[e^x]$

$=e^x\cdot e^x+e^x\cdot e^x=e^{2x}+e^{2x}$

$$\boxed{f'(x)=2e^{2x}}$$

Note: $\boxed{\frac{d}{dx}[e^{2x}]=2e^{2x}}$

5. $g(x)=e^{-x}=\frac{1}{e^x}$, $g'(x)=\frac{\frac{d}{dx}[1]\cdot e^x-1\cdot\frac{d}{dx}[e^x]}{(e^x)^2}=\frac{0-e^x}{(e^x)^2}$

$$\boxed{g'(x)=-\frac{e^x}{(e^x)}=-e^{-x}}$$

$\boxed{\frac{d}{dx}[e^{-x}]=-e^{-x}}$ There's a pattern!