MAT 136 (Calculus I), Prof. Jim Swift
In-Class Worksheet: The Chain Rule

Worth 5 class points. You may work in groups Name: key

1. Chain Rule using the Leibnitz notation $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Let $y = \sin(x^2)$. We can think of this as $y = \sin(u)$, where $u = x^2$.

(a) Compute $\frac{dy}{du}$ and $\frac{du}{dx}$. $\frac{dy}{du} = \cos(u)$ $\frac{du}{dx} = 2x$

(b) Compute $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$ in terms of u and x and then substitute in $u = x^2$ to get $\frac{dy}{dx}$ in terms of x alone.

$$\frac{dy}{dx} = \cos(u)\cdot 2x = \boxed{\cos(x^2)\cdot 2x}$$

2. Chain Rule using Newton's Notation $\frac{d}{dx}f(g(x)) = f'(g(x))\cdot g'(x)$

The function $y = \sin(x^2)$ is a composition of functions, $y = f(g(x))$, with

$f(u) =$ $\sin(u)$ and $g(x) =$ x^2 . Compute the derivatives of f and g:

$f'(u) =$ $\cos(u)$ and $g'(x) =$ $2x$. Practice the "eff of ex" notation:

$f'(x) =$ $\cos(x)$, $f'(y) =$ $\cos(y)$, $f'(3u) =$ $\cos(3u)$ $f'(x^2) =$ $\cos(x^2)$

Now evaluate the derivative, using the chain rule: $\frac{d}{dx}f(g(x)) = f'(g(x))\cdot g'(x)$

$\frac{d}{dx}\sin(x^2) =$ $\cos(x^2)\cdot 2x$

3. Let $y = h(x) = (x^2+3)^2$. Compute $\dfrac{dy}{dx} = h'(x)$ in two ways:

(a) By expanding $h(x)$ to write it as a polynomial in standard form and then differentiating with the "old" rules.

$h(x) = (x^2)^2 + 2(x^2)\cdot 3 + 3^2 = x^4 + 6x^2 + 9$

$\boxed{h'(x) = 4x^3 + 12x}$

(b) Using the chain rule. (You may use Leibnitz notation or Newton notation.)

Leibnitz:

$y = u^2, \quad u = x^2+3$

$\dfrac{dy}{du} = 2u, \quad \dfrac{du}{dx} = 2x$

so

$\dfrac{dy}{dx} = 2u \cdot 2x = 4ux$

$\boxed{\dfrac{dy}{dx} = 4(x^2+3)\cdot x}$

Newton

$h(x) = f(u), \quad u = x^2+3 = g(x)$

$f(u) = u^2, \ f'(u) = 2u$

so $f'(g(x)) = 2(x^2+3)$

$g'(x) = 2x$

so

$h'(x) = f'(g(x)) \cdot g'(x)$

$h'(x) = 2(x^2+3)\cdot 2x$

$\boxed{h'(x) = 4x(x^2+3)}$

(c) Show that you got the same answer in (b) and (c).

Factor (a) or expand (b).

$h'(x) = 4x^3 + 12x = 4x(x^2+3)$

factor out $4x$

$h'(x) = 4x(x^2+3) = 4x\cdot x^2 + 4x\cdot 3$

$= 4x^3 + 12x$

4. Differentiate $y = h(x) = (x+1)^{10}$. Note: One of the methods like 3(a) or 3(b) is very very much work.

$y = u^{10}, \quad u = (x+1)$

$\dfrac{dy}{du} = 10u^9, \quad \dfrac{du}{dx} = 1$, so $\boxed{\dfrac{dy}{dx} = 10(x+1)^9}$

Note: $(x+1)^{10} = x^{10} + 10x^9 + 45x^8 + 120x^7 + 210x^6 + 252x^5 +$

$+ 210x^4 + 120x^3 + 45x^2 + 10x + 1.$

You don't want to do it this way!