MAT 136 (Calculus I), Prof. Jim Swift
In-Class Worksheet: Derivative of logs, and implicit differentiation

1. Find the derivative of these functions:

$f(x) = \ln(3x+2)$ $f'(x) = \frac{1}{3x+2} \cdot \frac{d}{dx}(3x+2) = \frac{1}{3x+2} \cdot 3 = \boxed{\frac{3}{3x+2}}$

$g(x) = \ln|3x^2 - 2x - 1|$ $g'(x) = \frac{1}{3x^2-2x-1} \cdot \frac{d}{dx}(3x^2-2x-1) = \boxed{\frac{6x-2}{3x^2-2x-1}}$

$h(x) = \ln\left(\sqrt{\frac{|x|}{x^2+1}}\right)$. (Hint: Use logarithm identities to rewrite h, then differentiate.)

$h(x) = \frac{1}{2}\ln\left(\frac{|x|}{x^2+1}\right) = \frac{1}{2}(\ln|x| - \ln(x^2+1))$; $h'(x) = \boxed{\frac{1}{2}\left(\frac{1}{x} - \frac{2x}{x^2+1}\right)} = \boxed{\frac{1-x^2}{2x(x^2+1)}}$

2. Consider the curve in the x-y plane that satisfies $\sin(x) + \sin(xy) + y^2 = 1$. Do implicit differentiation and solve for $\frac{dy}{dx}$. See Dosmos.

$$\frac{d}{dx}\left[\sin(x) + \sin(xy) + y^2\right] = \frac{d}{dx}[1]$$

$$\cos(x) + \cos(xy) \cdot \frac{d}{dx}[xy] + 2y\frac{dy}{dx} = 0$$

Solve for $\frac{dy}{dx}$. See below for analogy:

$$\cos(x) + \cos(xy) \cdot \underbrace{\left(1 \cdot y + x\frac{dy}{dx}\right)} + 2y\frac{dy}{dx} = 0$$

$$\cos(x) + y\cos(xy) + x\cos(xy)\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$[\cos(x) + y\cos(xy)] + [x\cos(xy) + 2y]\frac{dy}{dx} = 0$$

Collect terms without $\frac{dy}{dx}$, and terms with $\frac{dy}{dx}$.

$$[x\cos(xy) + 2y]\frac{dy}{dx} = -[\cos(x) + y\cos(xy)]$$

$$\boxed{\frac{dy}{dx} = \frac{-(\cos(x) + y\cos(xy))}{x\cos(xy) + 2y}}$$

~~Re~~ analogy: Solve for m: $2 + 3(4 + 5m) + 6m = 0$

Collect terms without m, and terms with m.

$2 + 12 + 15m + 6m = 0$

$14 + 21m = 0$

$21m = -14$, $\boxed{m = \frac{-14}{21}}$