MAT 136 (Calculus I) Prof. Swift

In-class worksheet on Local Linearization: Feynman vs. the Abacus

Do this group work in groups of 3 or 4. When you finish, turn in *one* sheet per group, and do WeBWorK problems. key

First, read the story of Feynman vs. the Abacus. (Available on our web site, or via google search.) This worksheet will approximate $\sqrt[3]{1729.03}$ using a method equivalent to what Feynman did, but following the notation we use in our class. Use the facts, known by Feynman, that $12^2 = 144$ and $12^3 = 1728$.

1. (no technology) We want to compute $\sqrt[3]{1729.03}$. Define the function $f(x) = \sqrt[3]{x}$, so we want to approximate $f(1729.03)$. Feynman knew $f(1728) = \sqrt[3]{1728} =$ <u>12</u>. This is the first approximation for $\sqrt[3]{1729.03}$, and getting this approximation brought sweat to the forehead of the guy with the abacus.

2. (no technology) Note that $f(x) = x^{1/3}$. Compute $f'(x)$.

$$f'(x) = \frac{1}{3}x^{-2/3}$$

3. (no technology) Show that $f'(1728) = \dfrac{1}{3 \cdot 12^2}$. Hint: $(x^2)^3 = x^{2\cdot 3}$, and $(1728)^{\frac{1}{3}} = 12$

$$f'(1728) = \frac{1}{3}(1728)^{-2/3} = \frac{1}{3\cdot(1728)^{\frac{2}{3}}} = \frac{1}{3\cdot((1728)^{\frac{1}{3}})^2} = \frac{1}{3\cdot 12^2}$$

4. (no technology) To get a better approximation of $\sqrt[3]{1729.03}$, we will use the local linearization $f(x) \approx \ell_{1728}(x) = f(1728) + f'(1728) \cdot (x - 1728)$. Problems 1 and 3 say

$\ell_{1728}(x) =$ $12 + \frac{1}{3 \cdot 12^2}(x - 1728)$

5. (no technology) Now, approximate the cube root of 1729.03. Evaluate $1729.03 - 1728 = 1.03$ in your head, and get an exact expression for $\ell_{1728}(1729.03)$ involving a fraction with 1.03 in the numerator.

$\sqrt[3]{1729.03} \approx \ell_{1728}(1729.03) =$ $12 + \frac{1.03}{3 \cdot 12^2}$

6. (yes technology) Now, unlike Feynman, use technologogy to evaluate $\ell_{1728}(1729.03)$. Round to 9 significant figures. Use "..." for rounding, as in $\sqrt{2} = 1.41421356\ldots$.

$\sqrt[3]{1729.03} \approx$ 12.0023843...

7. (yes technology) Finally, find the true value of the cube root, rounded to 9 significant figures. (If you round to fewer than 9 significant figures, you get the same answer for 6 and 7.)

$\sqrt[3]{1729.03} =$ 12.0023838... (Both round to 12.002384...)

To relate this back to Feynman's story, he was able to approximate $\sqrt[3]{1729.03}$ by approximating $\ell_{1728}(1729) = 12 + \dfrac{1}{3 \cdot 12^2} = 12 + \dfrac{12}{3 \cdot 12^3} = 12 + \dfrac{4}{1728}$ without a calculator.