

MAT 136 (Calculus I), Prof. Jim Swift: Worksheet on the Shape of Graphs

Do this worksheet without the help of a calculator or computer.

Let the function f be defined by $f(x) = x^3 - 3x^2 + 1$. The domain of f is all real numbers.

(1) Find $f'(x)$ and $f''(x)$. $f'(x) = 3x^2 - 6x$, $f''(x) = 6x - 6$

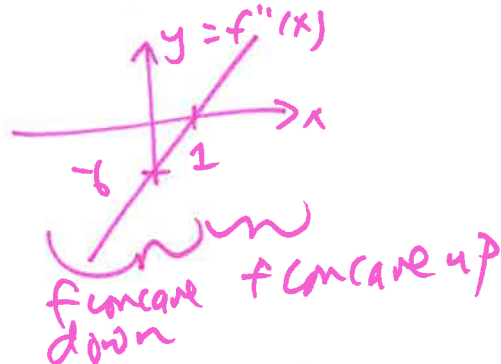
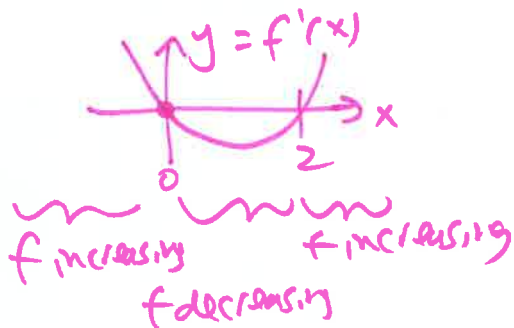
(2) Sketch the graphs of f' and f'' . Complete these sentences with $[a, b]$, $(-\infty, b]$, or $[a, \infty)$
 f is increasing and concave up on $[2, \infty)$ f is increasing and concave down on $(-\infty, 0]$
 f is decreasing and concave up on $[1, 2]$ f is decreasing and concave down on $[0, 1]$

(3) Find the turning points and inflection point of f . Recall that these are points in the (x, y) plane that are on the graph of f .

(4) Sketch the graph of f , indicating the turning points and inflection point. What important features of the graph cannot be calculated without a calculator or computer?

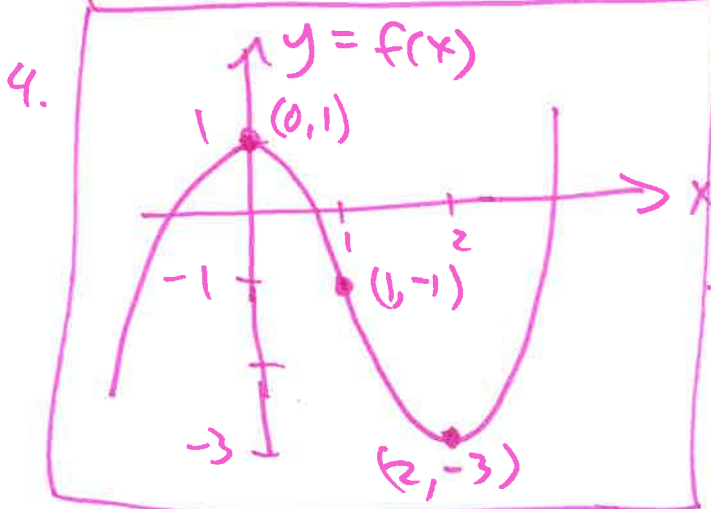
2. $f'(x) = 3x^2 - 6x = 3x(x-2)$

$f''(x) = 6x - 6 = 6(x-1)$



3. $f(0) = 1$, $f(1) = 1^3 - 3 \cdot 1^2 + 1 = 1 - 3 + 1 = -1$, $f(2) = 2^3 - 3 \cdot 2^2 + 1 = 8 - 12 + 1 = -3$

The turning points of f are $(0, 1)$ and $(2, -3)$.
 The inflection point of f is $(1, -1)$.



The x -intercepts of f cannot be calculated with pencil & paper.