MAT 136 (Calculus I) Prof. Swift

In-class worksheet: L'Hospital's Rule and Indeterminate Forms

1 a. (There are many correct answers.) Find two functions f and g such that

$$\lim_{x\to\infty} f(x) = \infty, \quad \lim_{x\to\infty} g(x) = \infty, \text{ and } \lim_{x\to\infty} (f(x) - g(x)) = 4.$$

$f(x) = x+4$, $g(x) = x$ is one pair.

1 b. (There are many correct answers.) Find two functions f and g such that

$$\lim_{x\to\infty} f(x) = \infty, \quad \lim_{x\to\infty} g(x) = \infty, \text{ and } \lim_{x\to\infty} (f(x) - g(x)) = \infty.$$

$f(x) = x^2$, $g(x) = x$ is one pair

2. Evaluate these two limits, using L'Hospital's rule if appropriate

$$\lim_{x\to 0} \frac{\cos(x) - 1}{x^2}, \qquad \lim_{x\to 0} \frac{\cos(x)}{x^2 - 1} = \frac{\cos(0)}{0^2-1} = \frac{1}{-1} = \boxed{-1}$$

Type $\frac{0}{0}$ "type $\frac{1}{-1}$" NOT an indeterminate form!

$$\lim_{x\to 0} \frac{\cos(x)-1}{x^2} \overset{L'H}{=} \lim_{x\to 0} \frac{\frac{d}{dx}\cos(x)-1}{\frac{d}{dx}(x^2)} = \lim_{x\to 0} \frac{-\sin(x)}{2x} \overset{!'H}{=} \lim_{x\to 0} \frac{-\cos(0)}{2} = \boxed{\frac{-1}{2}}$$

Type $\frac{0}{0}$

you may skip this step.