MAT 136 (Calculus I) Prof. Swift

In-Class Worksheet: The Definite Integral, Part 2

In this worksheet you will evaluate $\int_0^1 x\,dx$ in two ways.

1. Evaluate $\int_0^1 x\,dx$ using the Geometric Definition.

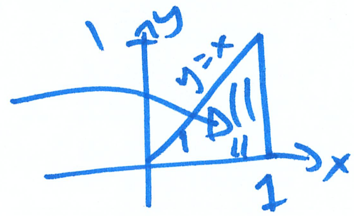
2. Find the Right Sum that approximates $\int_0^1 x\,dx$, with an arbitrary value of n.

Hints: (1) $R_n = \sum_{n=1}^{n} f(x_i)\Delta x$.

(2) As Gauss supposedly discovered, $1 + 2 + \ldots + n = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

3. Use the Limit Definition with right endpoints to evaluate $\int_0^1 f(x)\,dx = \lim_{n\to\infty} R_n$

1. $\int_0^1 x\,dx = A$



So $\boxed{\int_0^1 x\,dx = \frac{1}{2}}$

$A = \frac{1\cdot 1}{2} = \frac{1}{2}$

2. $a=0, b=1$, so $\Delta x = \frac{b-a}{n} = \frac{1-0}{n} = \frac{1}{n}$ and $x_i = 0 + i\Delta x = \frac{i}{n}$

$f(x_i) = x_i = \frac{i}{n}$, so $R_n = \sum_{n=1}^{n} f(x_i)\Delta x = \sum_{n=1}^{n} \frac{i}{n}\cdot\frac{1}{n}$

$R_n = \sum_{i=1}^{n} \frac{i}{n^2} = \frac{1}{n^2}\sum_{i=1}^{n} i = \frac{1}{n^2}\,\frac{n(n+1)}{2} = \boxed{\frac{n(n+1)}{2n^2}}$

So $\int_0^1 x\,dx = \lim_{n\to\infty} \frac{n(n+1)}{2n^2} = \lim_{n\to\infty} \frac{\frac{n}{n}\cdot\frac{n+1}{n}}{2} = \lim_{n\to\infty} \frac{1\cdot(1+\frac{1}{n})}{2}$

$= \lim_{n\to\infty} \frac{(1+\frac{1}{n})}{2} = \frac{1+0}{2} = \boxed{\frac{1}{2}}$.

Or, use L'Hôpital's Rule. $\int_0^1 x\,dx = \lim_{n\to\infty} \underbrace{\frac{n^2+n}{2n^2}}_{\text{Type } \frac{\infty}{\infty}} \overset{LH}{=} \lim_{n\to\infty} \underbrace{\frac{2n+1}{4n}}_{\text{Type } \frac{\infty}{\infty}} \overset{LH}{=} \lim_{n\to\infty} \frac{2}{4} = \frac{1}{2}$