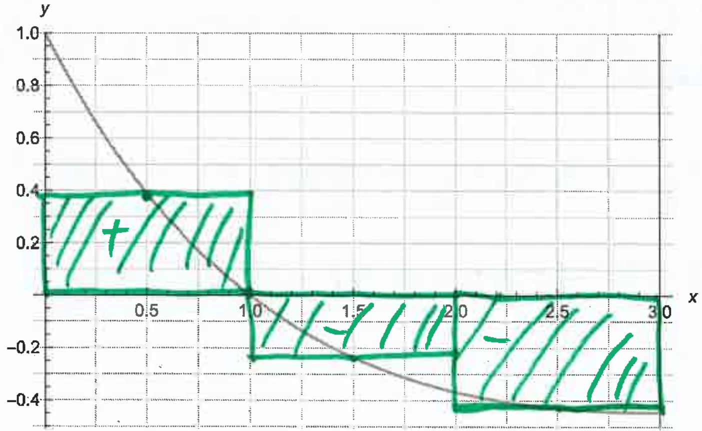
MAT 136 (Calculus I) Prof. Swift

In-class worksheet: The Definite Integral

The graph of a function f is shown. Approximate $\int_0^3 f(x)\,dx$ by estimating M_3, the midpoint rule with 3 subdivisions. Before writing down any numbers, draw rectangles to represent the areas you add or subtract to compute M_3. On the back of this sheet, estimating M_6 to approximate the same integral. Estimate the heights to 2 sig. figures.



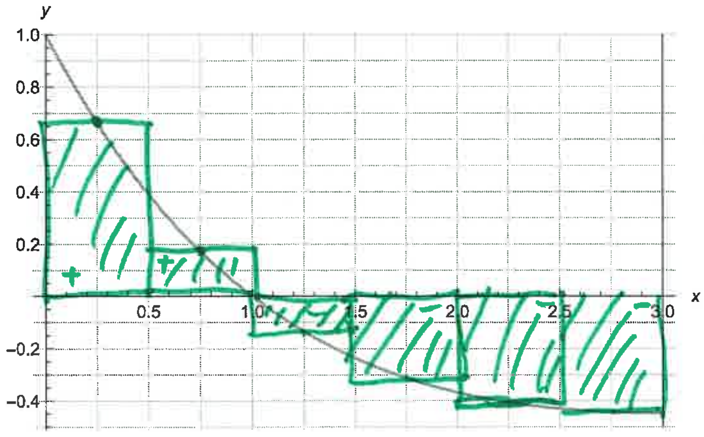
$M_3 = 0.39\times1 - 0.24\times1 - 0.43\times1$

$= 0.39 - 0.24 - 0.43$

$= -0.28$

So

$\int_0^3 f(x)\,dx \approx -0.28$



Note: The "true" integral is

$$\int_0^3 f(x)\,dx = -0.215\ldots = \lim_{n\to\infty} M_n$$

$M_3 = -0.28$, $M_6 = -0.23$

$$M_6 = 0.66\times\tfrac{1}{2} + 0.17\times\tfrac{1}{2} - 0.13\times\tfrac{1}{2} - \underbrace{\cancel{0.41}}_{0.31}\times\tfrac{1}{2} - 0.41\times\tfrac{1}{2} - 0.44\times\tfrac{1}{2}$$

$$= (0.66 + 0.17 - 0.13 - 0.31 - 0.41 - 0.44)\times\tfrac{1}{2}$$

$$= (-0.46)\times\tfrac{1}{2}$$

$\boxed{M_6 = -0.23}$, so $\int_0^3 f(x)\,dx \approx -0.23$