MAT 136 (Calculus I) Prof. Swift

In-class worksheet: The method of u-substitution, part 2

1. Evaluate the indefinite integral.

$$\int \frac{x+1}{x^2+1}\,dx = \int \frac{x}{x^2+1}\,dx + \int \frac{1}{x^2+1}\,dx = \frac{1}{2}\int \frac{1}{u}\,du + \arctan(x) + C$$

$u = x^2+1$, $du = 2x\,dx$ in first integral

By inspection in second.

$$= \frac{1}{2}\ln|u| + \arctan(x) + C$$

$$= \boxed{\frac{1}{2}\ln(x^2+1) + \arctan(x) + C}$$

2. What is the value of b that makes this equality true?

$$\int_0^{\sqrt{\pi}} 2x\sin(x^2)\,dx = \int_0^b \sin(u)\,du$$

$u = x^2$, $du = 2x\,dx$

limits: $x = \sqrt{\pi} \Rightarrow u = (\sqrt{\pi})^2 = \pi$ $\therefore \boxed{b = \pi}$

$x = 0 \Rightarrow u = 0^2 = 0$

3. The sine integral function is defined as $\mathrm{Si}(x) = \displaystyle\int_0^x \frac{\sin(t)}{t}\,dt$. Write the integral $\displaystyle\int_0^2 \frac{\sin(\pi x)}{x}\,dx$ in terms of the Si function.

$u = \pi x$

$du = \pi\,dx$

$x = \frac{u}{\pi}$

$dx = \frac{1}{\pi}du$

$$\int_0^2 \frac{\sin(\pi x)}{x}\,dx = \int_0^{2\pi} \frac{\sin(u)}{\left(\frac{u}{\pi}\right)} \cdot \frac{1}{\pi}\,du$$

limits: $x = 2 \Rightarrow u = 2\pi$

$x = 0 \Rightarrow u = 0$

$$= \int_0^{2\pi} \frac{\sin(u)}{u}\,du$$

$$= \boxed{\mathrm{Si}(2\pi)}$$