Theorem/Def.
$$\int_a^b f(x)\,dx = \lim_{n\to\infty} \sum_{i=1}^{n} \underline{f(x_i^*)\Delta x}$$

$f(x_i^*)\Delta x = F(x_i) - F(x_{i-1})$ for cleverly chosen x_i^*

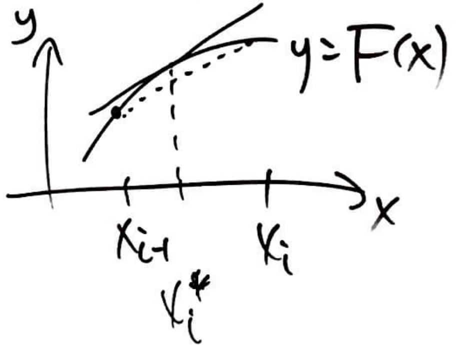
$\Delta x = \frac{b-a}{n}$, $x_i = a + i\Delta x$ (so $x_n = b$), and x_i^* is <u>any</u> sample point in the interval $[x_{i-1}, x_i]$ $\quad x_{i-1} \le x_i^* \le x_i$

FToC: $\int_a^b f(x)\,dx = F(b) - F(a)$, where $F' = f$.

"Proof": we can choose those x_i^* so $\sum_{i=1}^{n} f(x_i^*)\Delta x = F(b) - F(a)$.

Mean Value Theorem. Apply to $F(x)$ on the interval $[x_{i-1}, x_i]$.

There exists x_i^* such that $F'(x_i^*) = \frac{F(x_i) - F(x_{i-1})}{x_i - x_{i-1}}$, $\quad f(x_i^*) = \frac{F(x_i) - F(x_{i-1})}{\Delta x}$



So, with x_i^* chosen just right

$$\sum_{i=1}^{n} f(x_i^*)\,\Delta x = \sum_{i=1}^{n} F(x_i) - F(x_{i-1})$$

$$= \underbrace{F(x_1) - F(x_0)}_{i=1} + \underbrace{F(x_2) - F(x_1)}_{i=2} + \cdots + \underbrace{F(x_{n-1}) - F(x_{n-2})}_{i=n-1} + \underbrace{F(x_n) - F(x_{n-1})}_{i=n}$$

cancel cancel

$$= \underbrace{F(x_n) - F(x_{n-1})}_{i=n} + \underbrace{F(x_{n-1}) - F(x_{n-2})}_{i=n-1} + \cdots + \underbrace{F(x_2) - F(x_1)}_{i=2} + \underbrace{F(x_1) - F(x_0)}_{i=1}$$

$$= F(x_n) - F(x_0)$$

$$= F(b) - F(a)$$