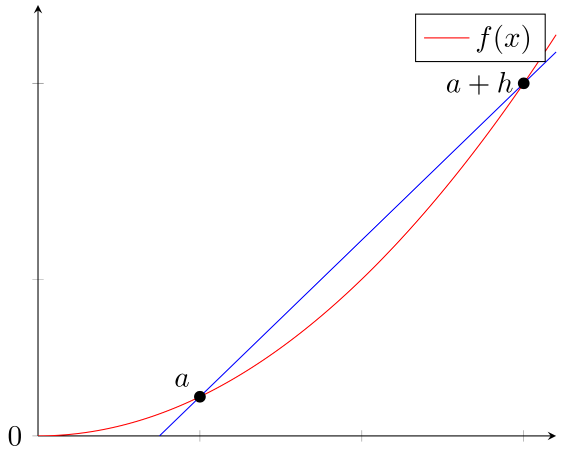
MAT 136 Week 4, Day 1: Limit Definition of the Derivative

The average rate of change of a function f from $x = a$ to $x = b$ is the __________ of the ________________ __________ to the graph of $y = f(x)$ that passes through ________________ and ________________.

The **derivative** of the function f at the point $x = a$ is the ______________________ __________ of __________ at $x = a$.

It is also the slope of the tangent to the graph of $f(x)$ at the point $(a, f(a))$. We looked at this scenario in Week 1, Day 4.



To find the slope of the tangent at $(a, f(a))$, we can start with a secant through another point on the graph (say, $(a + h, f(a + h))$), and then let h approach 0. Now that we have limits, we can do this formally.

Definition. Let f be a function and let $a \in \mathbb{R}$. The **derivative** of f at $x = a$, if it exists, is

Sometimes the slope of the secant line to $y = f(x)$ going through the points $(a, f(a))$ and $(a + h, f(a + h))$ is called a ______________________ ______________________.

Example 1. Consider the function $f(x) = -2x+5$. For each real number a, determine the derivative of f at the point a.

If $f(x) = mx + b$ (where m, b are constants), then for each a the derivative of f at a is equal to .

Example 2. Find an equation for the tangent line to the graph $y = x^2$ at $x = 5$.

Example 3. The controls on your remote control car are malfunctioning, causing your car to accelerate linearly. As a result, the current position $p(t)$ of your car from your starting point (in feet) after t seconds can be modelled by $p(t) = t^3 + 8t$. Determine the instantandeous velocity of your car at $t = 2$ seconds.

Example 4. Find a formula for the instantaneous rate of change of the function $f(x) = \frac{1}{x}$ at the point $x = a$, in terms of a. (Assume that $a \neq 0$.)