MAT 136 (Calculus I), Prof. Jim Swift
The Point-Slope Form of Linear Functions, and Inverse Functions

1. In this problem, we consider the conversion of temperature from Celsius to Fahrenheit. This is useful if you travel to Europe, or in fact almost anywhere outside of the USA.

Let $F = f(C)$ be the temperature in Fahrenheit as a function of the temperature C in Celsius. We know that water freezes at $C = 0$ and $F = 32$, and that water boils at $C = 100$ and $F = 212$. Thus, $f(0) = 32$ and $f(100) = 212$.

(a) Find the slope of the function f. Hint: the slope is $m = \frac{\Delta F}{\Delta C}$. Use decimals. $m = \frac{212-32}{100-0} = \frac{180}{100}$

$m = 1.8$

(b) Compute the change in F when C changes by 10. Use that to fill in the following table:

C	0	10	20	30	40	50
F	32	50	68	86	104	122

$\Delta F = 10m = 18$ when $\Delta C = 10$

(c) Use the slope m to fill in the table with only addition or subtraction, but no multiplication.

C	19	20	21	22
F	66.2	68	69.8	71.6

$f(C+1) = f(C) + 1.8$

(d) Find the formula for F as a function of C using the slope-intercept form: $F = f(C) = mC + b$. That is, fill in the blanks: $F = 1.8\,C + 32$

(e) In part (c), you found the value of $f(C)$ near $C = 20$ using the point-slope form of f, based on $f(20) = 68$. Fill in the blanks using that way to write the function: $F = 1.8\,(C-20) + 68$.

2. Find the formula for C as a functon of F. This is $C = f^{-1}(F)$. In other words, find ~~the~~ a formula for the inverse function f^{-1}.

There are many correct answers.

$F = 1.8C + 32$

$F - 32 = 1.8C = \frac{9}{5}C$ (or)

$C = \frac{5}{9}(F-32)$

$F = 1.8(C-20) + 68$

$F - 68 = \frac{9}{5}(C-20)$

$C - 20 = \frac{5}{9}(F-68)$

$C = \frac{5}{9}(F-68) + 20$

or

$f^{-1}(F) = \frac{5}{9}(F-32)$

$f^{-1}(F) = \frac{5}{9}(F-68) + 20$

Note: (Bonus) $\frac{5}{9} = 0.555\ldots$

If $F = 69$, then

$C = f^{-1}(69) = \frac{5}{9} + 20 = 20.555\ldots$