MAT 136 (Calculus I), Prof. Jim Swift
In-Class Worksheet: The Chain Rule

Worth 5 class points. You may work in groups Name: key.

These marks are required.

These are extra explanation.

1. Chain Rule using the Leibnitz notation $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$

Let $y = \sin(x^2)$. We can think of this as $y = \sin(u)$, where $u = x^2$.

(a) Fill in the blanks. $\frac{dy}{du} = \cos(u)$ and $\frac{du}{dx} = 2x$

(b) Compute $\frac{dy}{dx} = \frac{dy}{du}\frac{du}{dx}$ in terms of u and x and then substitute in $u = x^2$ to get $\frac{dy}{dx}$ in terms of x alone.

$$\frac{dy}{dx} = \cos(u) \cdot 2x = \cos(x^2) \cdot 2x$$

2. Chain Rule using Newton's Notation $\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$

The function $y = \sin(x^2)$ is a composition of functions, $y = f(g(x))$, with

$f(u) = \sin(u)$ and $g(x) = x^2$. In the next line, compute the derivatives:

$f'(u) = \cos(u)$ and $g'(x) = 2x$. In the next line, practice "eff of ex" notation:

$f'(x) = \cos(x)$, $f'(y) = \cos(y)$, $f'(3u) = \cos(3u)$, $f'(x^2) = \cos(x^2) = f'(g(x))$

Now evaluate the derivative, using the chain rule: $\frac{d}{dx} f(g(x)) = f'(\overbrace{g(x)}) \cdot g'(x)$

$\frac{d}{dx} \sin(x^2) = \cos(x^2) \cdot 2x$

3. Find the derivative of $y = e^{-x^2+x}$, also written as $y = \exp(-x^2 + x)$, using both methods.

$\boxed{y = e^u, \text{ where } u = -x^2 + x}$ Need these! Later we will skip to answer.

$\frac{dy}{du} = e^u$, $\frac{du}{dx} = -2x + 1$

$$\boxed{\frac{dy}{dx} = e^u(-2x+1) = e^{-x^2+x}(-2x+1)}$$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$: Leibnitz

$\boxed{y = f(g(x)), \text{ where } f(u) = e^u, \text{ and } g(x) = -x^2 + x}$

$f'(u) = e^u$, $g'(x) = \cancel{-x}+ = -2x + 1$

$$\boxed{\frac{dy}{dx} = e^{-x^2+x}(-2x+1)}$$

$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$: Newton