MAT 136 (Calculus I), Prof. Jim Swift
In-Class Worksheet: Derivative Shortcuts 3.

This is a pencil-and-paper exercise. No calculators.

1. Let $f(x) = 3x^2 - 4x + 7$. Compute $f'(x)$, $f''(x)$, $f'''(x)$, and $f^{(2024)}(x)$.

$f'(x) = 6x - 4, \quad f''(x) = 6, \quad f'''(x) = 0, \quad f^{(2024)}(x) = 0$

2. Compute $\frac{d}{dx}[x^2 \sin(x)] = \frac{d}{dx}[x^2] \cdot \sin(x) + x^2 \frac{d}{dx}[\sin(x)] = \boxed{2x\sin(x) + x^2\cos(x)}$

3. Let $f(x) = \sec(x)$. Find $f'(x)$ using a trig identity and the quotient rule.

$$f(x) = \frac{1}{\cos(x)}, \text{ so } f'(x) = \frac{\overbrace{\frac{d}{dx}[1]}^{0} \cdot \cos(x) - 1 \cdot \frac{d}{dx}[\cos(x)]}{(\cos(x))^2}$$

$$= \frac{0 - 1 \cdot (-\sin(x))}{\cos^2(x)} = \boxed{\frac{\sin(x)}{\cos^2(x)}}$$

Another form of the answer is

$$\frac{d}{dx}[\sec(x)] = \frac{\sin(x)}{\cos^2(x)} = \frac{\sin(x)}{\cos(x)} \cdot \frac{1}{\cos(x)} = \tan(x) \cdot \sec(x)$$

you may want to memorize this fact, but you don't need to for me:

$$\boxed{\frac{d}{dx}[\sec(x)] = \tan(x)\sec(x)}$$