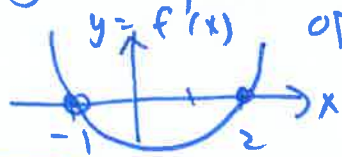
1. Find the largest interval(s) on which $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x$ is increasing.

$f'(x) = x^2 - x - 2 = (x+1)(x-2)$. graph of $f'(x) = (x+1)(x-2)$ opens up



$f'(x) \geq 0$ on $(-\infty, -1]$ and $f'(x) = 0$ only at $x = -1$ (and $x = 2$)

Therefore, $\boxed{f \text{ is increasing on } (-\infty, -1].}$

Similarly, $f'(x) \geq 0$ on $[2, \infty)$ and $f'(2) = 0$ is the only place where $f'(x) = 0$.

Therefore, $\boxed{f \text{ is increasing on } [2, \infty).}$

Note: It is NOT correct to write ~~f is increasing on $(-\infty, -1] \cup (2, \infty)$~~. False

NOT an interval

2. Find the largest interval(s) on which $g(x)=\frac{1}{3}x^3-x^2+x$ is increasing.

$g'(x)=x^2-2x+1=(x-1)^2$. graph of g':

$g'(x)\geq 0$ for all x, and $g'(x)=0$ has only one solution ($x=1$).

Therefore, $\boxed{g \text{ is increasing on } (-\infty,\infty).}$

Note: It is also true that g is increasing on each of these intervals:
$(-\infty,1)$, $(1,\infty)$, $(-\infty,1]$, $[1,\infty)$, $(2,3)$, in fact any interval.
But the largest interval on which g is increasing is $(-\infty,\infty)$.
Note that $(-\infty,1)\cup(1,\infty)$ is NOT an interval.