

# MAT 136 (Calculus I) Prof. Swift

## In-Class Worksheet: The First Derivative Test to Classify Critical Points

Consider the function  $f(x) = \frac{1}{3}x^3 + x^2 - 3x + 4$ .

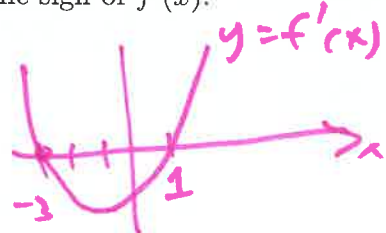
1. Compute the derivative  $f'(x)$ .  $f'(x) = x^2 + 2x - 3 = (x+3)(x-1)$

2. Find all of the critical points of  $f$ .  $f'(x) = 0$  has solutions  $C_1 = -3$  and  $C_2 = 1$

3. Sketch  $y = f'(x)$ . Only worry about the  $x$ -intercepts, and the sign of  $f'(x)$ .

4. Apply the first derivative test to each critical point of  $f$ .

5. Do steps 1-4 for  $g(x) = \frac{1}{3}x^3 - x^2 + x + 2$ .



$f$  has a local max at  $-3$ .

Since  $f'(x) > 0$  for  $x < -3$  and  $f'(x) < 0$  for  $x > -3$  (and  $x < 1$ ).

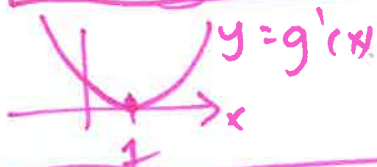
$f$  has a local min at  $1$ .

Since  $f'(x) < 0$  for  $x < 1$  (and  $x > -3$ ), and  $f'(x) > 0$  for  $x > 1$ .

5.  $g(x) = \frac{1}{3}x^3 - x^2 + x + 2$

$$g'(x) = x^2 - 2x + 1 = (x-1)^2$$

The only critical point is  $1$ . (i.e.,  $x=1$ ).



$g$  does not have a local extremum because

$g'(x)$  has the same sign (positive) for  $x < 1$  and for  $x > 1$ .