

**MAT 136 (Calculus I), Prof. Jim Swift:**  
**Worksheet on the Shape of Graphs**

Name: \_\_\_\_\_

You may work with others, but turn in your own paper. Do this worksheet without the help of a calculator or computer. You may use the back if needed.

Let the function  $f$  be defined by  $f(x) = x^3 - 3x^2 + 1$ . The domain of  $f$  is all real numbers.

(1) Find  $f'(x)$  and  $f''(x)$ .

(2) Sketch the graphs of  $f'$  and  $f''$ . Complete these sentences with intervals written in the form  $(a, b)$ ,  $(-\infty, b)$ , or  $(a, \infty)$ .

$f$  is increasing and concave up on

$f$  is increasing and concave down on

$f$  is decreasing and concave up on

$f$  is decreasing and concave down on

(3) Find the critical points of  $f$ . Classify each as a local maximum or a local minimum using the second derivative test.

(4) Find the inflection point of  $f$ . Recall that this is a point in the  $(x, y)$  plane that is on the graph of  $f$ .

(5) Sketch the graph of  $f$ , indicating the local extrema and the inflection point. What important features of the graph cannot be calculated without a calculator or computer?