MAT 136 (Calculus I), Prof. Jim Swift: Worksheet on the Shape of Graphs

Name: key

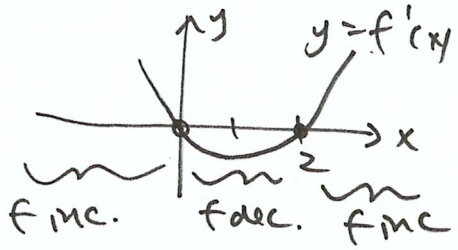
You may work with others, but turn in your own paper. Do this worksheet without the help of a calculator or computer. You may use the back if needed.

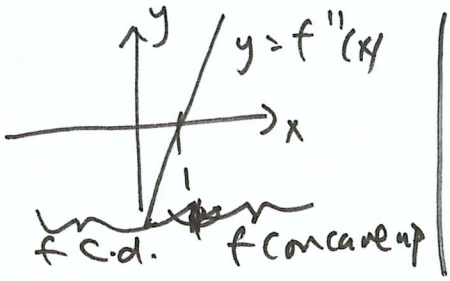
Let the function f be defined by $f(x) = x^3 - 3x^2 + 1$. The domain of f is all real numbers.

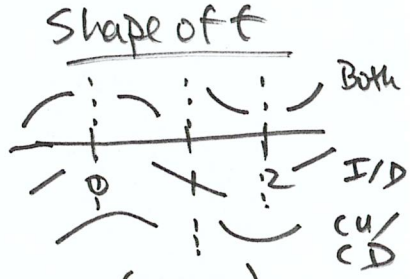
(1) Find $f'(x)$ and $f''(x)$. $f'(x) = 3x^2 - 6x = 3x(x-2)$

$f''(x) = 6x - 6 = 6(x-1)$

(2) Sketch the graphs of f' and f''. Complete these sentences with intervals written in the form (a, b), $(-\infty, b)$, or (a, ∞).







f is increasing and concave up on $(2, \infty)$

f is increasing and concave down on $(-\infty, 0)$

f is decreasing and concave up on $(1, 2)$

f is decreasing and concave down on $(0, 1)$

(3) Find the critical points of f. Classify each as a local maximum or a local minimum using the second derivative test.

$f'(x) = 0$ is $3x(x-2) = 0$

So $x = 0$ and $x = 2$ are the critical points.

$f''(0) = -6 < 0$, so ~~$x = 0$ is a local~~, so f has a local max at $x = 0$

$f''(2) = 6 > 0$, so f has a local min at $x = 2$.

(4) Find the inflection point of f. Recall that this is a point in the (x, y) plane that is on the graph of f.

$f''(x) = 0$ is $6(x-1) = 0$

$x = 1$.

$f(1) = 1^3 - 3 \cdot 1^2 + 1 = 1 - 3 + 1 = -1$

So $(1, -1)$ is the inflection point of f.

(5) Sketch the graph of f, indicating the local extrema and the inflection point. What important features of the graph cannot be calculated without a calculator or computer? the x-intercepts.

Note: $f(0) = 1$, so $(0, 1)$ is on graph of f

$f(2) = 2^3 - 3 \cdot 2^2 + 1 = 8 - 12 + 1 = -3$, so $(2, -3)$ is on graph of f.

Note: $f(3) = 3^3 - 3 \cdot 3^2 + 1 = 1$

$f(-1) = (-1)^3 - 3(-1)^2 + 1$

$= -1 - 3 + 1 = -3$