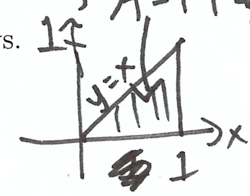
MAT 136 (Calculus I) Prof. Swift

In-Class Worksheet: The Definite Integral, Part 2

In this worksheet you will evaluate $\int_0^1 x\,dx$ in two ways.

$A = 1 \cdot 1 \cdot \frac{1}{2} = \frac{1}{2}$



So $\boxed{\int_0^1 x\,dx = \frac{1}{2}}$

1. Evaluate $\int_0^1 x\,dx$ using the Geometric Definition.

2. Find the Right Sum that approximates $\int_0^1 x\,dx$, with an arbitrary value of n.

Hints: (1) $R_n = \sum_{i=1}^{n} f(x_i)\Delta x$.

(2) As Gauss supposedly discovered, $1 + 2 + \ldots + n = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}$.

3. Use the Limit Definition with right endpoints to evaluate $\int_0^1 f(x)\,dx = \lim_{n\to\infty} R_n$

2. $\Delta x = \frac{1-0}{n} = \frac{1}{n}$, and $x_i = 0 + i\Delta x$, so $x_i = \frac{i}{n}$

$R_n = \sum_{i=1}^{n} f(x_i)\Delta x = \sum_{i=1}^{n} (x_i) \cdot \frac{1}{n} = \sum_{i=1}^{n} \frac{i}{n} \cdot \frac{1}{n} = \sum_{i=1}^{n} \frac{i}{n^2}$

$R_n = \frac{1}{n^2} \sum_{i=1}^{n} i$, since n is a constant in the sum

$R_n = \frac{1}{n^2} \cdot \frac{n(n+1)}{2}$, using hint (2)

$\boxed{R_n = \frac{n+1}{n \cdot 2}}$ simplified form.

3. $\int_0^1 x\,dx = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \frac{n+1}{2n} \overset{L'H}{=} \lim_{n\to\infty} \frac{1}{2} = \boxed{\frac{1}{2}}$

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Note: $\boxed{R_n = \frac{1}{2} + \frac{1}{2n}}$ is another simplified form.