

MAT 136 (Calculus I) Prof. Swift

In-class worksheet: The Fundamental Theorem of Calculus, Part 1

1. Evaluate $\int_0^2 x^3 dx = \frac{x^4}{4} \Big|_0^2 = \frac{2^4}{4} - \frac{0^4}{4} = \frac{16}{4} = \boxed{4}$

2. Evaluate $\int_0^3 t^2 dt = \frac{t^3}{3} \Big|_0^3 = \frac{3^3}{3} - \frac{0^3}{3} = 3^2 = \boxed{9}$

3. Evaluate $\int_0^x t^2 dt = \frac{t^3}{3} \Big|_0^x = \frac{x^3}{3} - \frac{0^3}{3} = \boxed{\frac{x^3}{3}}$

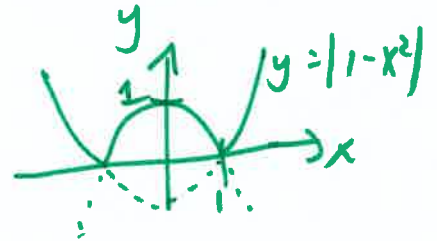
4. Evaluate $\int_0^2 |1 - x^2| dx$. Hint: $\int_a^b f(x) dx + \int_b^c f(x) dx = \int_a^c f(x) dx$

Recall: $|x| = \begin{cases} x, & \text{if } x \geq 0 \\ -x, & \text{if } x < 0 \end{cases}$



So $|1 - x^2| = \begin{cases} 1 - x^2 & \text{if } 1 - x^2 \geq 0 \\ -(1 - x^2) & \text{if } 1 - x^2 < 0 \end{cases}$

$$= \begin{cases} x^2 - 1 & \text{if } x < -1 \\ 1 - x^2 & \text{if } -1 \leq x \leq 1 \\ x^2 - 1 & \text{if } x > 1 \end{cases}$$



So $\int_0^2 |1 - x^2| dx = \int_0^1 |1 - x^2| dx + \int_1^2 |1 - x^2| dx$

$$= \int_0^1 (1 - x^2) dx + \int_1^2 (x^2 - 1) dx$$

$$= \left(x - \frac{x^3}{3}\right) \Big|_0^1 + \left(\frac{x^3}{3} - x\right) \Big|_1^2$$

$$= 1 - \frac{1^3}{3} - (0 - \frac{0^3}{3}) + \left(\frac{2^3}{3} - 2\right) - \left(\frac{1^3}{3} - 1\right)$$

$$= 1 - \frac{1}{3} + \frac{8}{3} - 2 + 1 - \frac{1}{3} = 1 - 2 + 1 + \frac{8}{3} - \frac{2}{3}$$

$$= \frac{6}{3} = \boxed{2}$$

Thus,

$$\int_0^2 |1 - x^2| dx = 2$$