Series Convergence and Divergence Tests 2023-11-08 version		
Partial Sums	$s_n = \sum_{i=1}^{n} a_i$	$\sum_{n=1}^{\infty} a_n$ converges if and only if $\lim_{n\to\infty} s_n$ exists. $\sum_{n=1}^{\infty} a_n = \lim_{n\to\infty} s_n$.
Test for Divergence	Apply this test first.	If $\lim_{n\to\infty} a_n \neq 0$, then $\sum_{n=1}^{\infty} a_n$ diverges. Never proves convergence!
Geometric Series	$a + ar + ar^2 + \cdots$ $s_n = a + ar + \cdots + ar^{n-2} + ar^{n-1}$	If $\lvert r\rvert < 1$, then series converges to $\frac{a}{1-r} = \frac{\text{first term}}{1 - \text{common ratio}}$. If $\lvert r\rvert \geq 1$, then the series diverges. $s_n = \frac{a - ar^n}{1-r} = \frac{\text{first term} - \text{first missing term}}{1 - \text{common ratio}}$
p-Series	$\sum_{n=1}^{\infty} \frac{1}{n^p}$	If $p > 1$ the p-series converges. If $p \leq 1$ the p-series diverges.
Comparison Test	$\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$, where $0 \leq a_n \leq b_n$	If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges. If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.
Limit Comparison	For $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ where $a_n > 0$, $b_n > 0$, and $0 < \lim_{n\to\infty} \frac{a_n}{b_n} < \infty$	$\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ both converge or both diverge.
Integral Test	$\sum_{n=1}^{\infty} a_n$ where $f(n) = a_n$ and f is continuous, positive, and decreasing.	If $\int_1^{\infty} f(x)dx$ converges, then $\sum_{n=1}^{\infty} a_n$ converges. If $\int_1^{\infty} f(x)dx$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges.
Alternating Series	$\pm \sum_{n=1}^{\infty} (-1)^n a_n$	If $a_n > 0$ is decreasing and $\lim_{n\to\infty} a_n = 0$, the series converges.
Absolute Convergence	a_n not all same sign	If $\sum_{n=1}^{\infty} \lvert a_n\rvert$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.
Ratio Test	$L = \lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert$	If $L < 1$, then $\sum_{n=1}^{\infty} a_n$ converges absolutely, and thus converges. If $L = 1$, then the ratio test is inconclusive. If $L > 1$, or $\lim_{n\to\infty} \left\lvert \frac{a_{n+1}}{a_n} \right\rvert = \infty$, then $\sum_{n=1}^{\infty} a_n$ diverges.