MAT 137 (Calculus II) Prof. Swift

Exam 1, 10:20-11:10, February 8, 2006

Review and Chapter 5

Name: key

Code name for posting grades on the web: possible

No notes are allowed. No calculators are allowed.

All problems have equal weight.

The exam will begin and end promptly.

1. Find the derivatives.

(a) $f(x) = x^3 + 3x^2 + 5$

$f'(x) = 3x^2 + 6x$

(b) $y = \sin(e^x)$

$y' = \cos(e^x) \cdot e^x$

Simplify your expression for $g'(x)$ in the next part.

(c) $g(x) = \dfrac{x+1}{x-1}$ $\quad g'(x) = \dfrac{(x+1)'(x-1) - (x+1)(x-1)'}{(x-1)^2}$

$= \dfrac{(x-1) - (x+1)}{(x-1)^2} = \boxed{\dfrac{-2}{(x-1)^2}}$

(d) $f(x) = \cos(x) \cdot (x^2 + x + 1)$

$f'(x) = -\sin(x) \cdot (x^2+x+1) + \cos(x) \cdot (2x+1)$

(e) $g(x) = \arcsin(x^2)$ $\quad g'(x) = \dfrac{1}{\sqrt{1-(x^2)^2}} \cdot 2x = \dfrac{2x}{\sqrt{1-x^4}}$

(f) $y = \ln|x^2 - 1|$ $\quad y' = \dfrac{1}{x^2-1} \cdot 2x = \dfrac{2x}{x^2-1}$

2. Compute the integrals.

(a) $\int (x^2 + x + 1)\, dx = \frac{x^3}{3} + \frac{x^2}{2} + x + C$

(b) $\int \left(\sin(x) + 2\cos(x) + \frac{3}{1+x^2}\right) dx = -\cos(x) + 2\sin(x) + 3\arctan(x) + C$

(c) $\int x e^{x^2} dx = \frac{1}{2}\int e^{x^2} \cdot 2x\, dx = \frac{1}{2}\int e^u du$

$u = x^2$

$du = 2x\, dx$

$= \frac{1}{2} e^u + C = \boxed{\frac{1}{2} e^{x^2} + C}$

3. Compute the integrals.

(a) $\int \frac{x^2 - 3x + 2}{x} dx = \int \left(x - 3 + \frac{2}{x}\right) dx$

Simplify integrand.

$= \frac{x^2}{2} - 3x + 2\ln|x| + C$ } either one is OK

$= \frac{x^2}{2} - 3x + \ln(x^2) + C$ }

(b) $\int \frac{x}{(x-1)(x-2)} dx = \int \left(\frac{-1}{x-1} + \frac{2}{x-2}\right) dx = \boxed{-\ln|x-1| + 2\ln|x-2| + C}$

Partial Fractions

$\frac{x}{(x-1)(x-2)} = \frac{A}{x-1} + \frac{B}{x-2}$

$A = \frac{1}{1-2} = \frac{1}{-1} = -1$; $B = \frac{2}{2-1} = 2$

This uses the trick I taught you.