MAT 136, Prof. Swift
Differentiation Shortcuts with MAT 137 Additions in Red

Rules:

$\frac{d}{dx}[c \cdot f(x)] = c \cdot f'(x)$ The Constant Multiple Rule (c is any constant). $(cf)' = cf'$

$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$ The Sum and Difference Rules. $(f \pm g)' = f' \pm g'$

$\frac{d}{dx}[f(x) \cdot g(x)] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$ The Product Rule. $(f \cdot s)' = f' \cdot s + f \cdot s'$

$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x) \cdot g(x) - f(x) \cdot g'(x)}{[g(x)]^2}$ The Quotient Rule. $\left(\frac{t}{b}\right)' = \frac{t' \cdot b - t \cdot b'}{b^2}$

$\frac{d}{dx} f(g(x)) = f'(g(x)) \cdot g'(x)$ The all-important Chain Rule. $(f \circ g)' = (f' \circ g) \cdot g'$

$\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}$ The all-important Chain Rule in Leibnitz notation.

Facts:

$\frac{d}{dx}[m \cdot x + b] = m$ for any constants m and b. This implies $\frac{d}{dx} c = 0$, so $\frac{d}{dx}[f(x) + c] = f'(x)$.

$\frac{d}{dx} x^c = c \cdot x^{c-1}$ for any constant $c \neq 0$ (positive, negative, integer, fraction, or irrational).

$\frac{d}{dx} e^x = e^x$ and more generally $\frac{d}{dx} a^x = \ln(a) \cdot a^x$ for any positive constant a.

$\frac{d}{dx} \sin(x) = \cos(x)$ $\frac{d}{dx} \cos(x) = -\sin(x)$ $\frac{d}{dx} \tan(x) = \frac{1}{\cos^2(x)} = \sec^2(x)$

$\frac{d}{dx} \sinh(x) = \cosh(x)$ $\frac{d}{dx} \cosh(x) = \sinh(x)$ $\frac{d}{dx} \tanh(x) = \frac{1}{\cosh^2(x)} = \text{sech}^2(x)$

$\frac{d}{dx} \ln(x) = \frac{1}{x}$ with domain $x > 0$ $\frac{d}{dx} \ln|x| = \frac{1}{x}$ with domain $x \neq 0$

$\frac{d}{dx} \arcsin(x) = \frac{1}{\sqrt{1 - x^2}}$ $\frac{d}{dx} \arctan(x) = \frac{1}{1 + x^2}$

The Trigonometric Identities you need to know:

$\sin^2(x) + \cos^2(x) = 1$ $\tan(x) = \frac{\sin(x)}{\cos(x)}$

To do some integrals, also memorize $\cos^2(x) = \frac{1}{2}(1 + \cos(2x))$ and $\sin^2(x) = \frac{1}{2}(1 - \cos(2x))$.

$\csc(x) = \frac{1}{\sin(x)}$ $\sec(x) = \frac{1}{\cos(x)}$ $\cot(x) = \frac{1}{\tan(x)} = \frac{\cos(x)}{\sin(x)}$

The Hyperbolic Trigonometric Identities you need to know:

$\cosh^2(x) - \sinh^2(x) = 1$ $\qquad$ $\tanh(x) = \dfrac{\sinh(x)}{\cosh(x)}$ $\qquad$ $\text{sech}(x) = \dfrac{1}{\cosh(x)}$

The Logarithm Identities you need to know:

$\ln(a) = b \Leftrightarrow e^b = a$, so $\ln(1) = 0$ (since $e^0 = 1$), and $\ln(e) = 1$ (since $e^1 = e$), etc.

$e^{\ln(x)} = x$ for all $x > 0$ $\qquad$ $\ln(e^x) = x$ for all x $\qquad$ $\log_a(x) = \ln(x)/\ln(a)$ $\qquad$ $a^x = e^{\ln(a)\cdot x}$

$\ln(ab) = \ln(a) + \ln(b)$ $\qquad$ $\ln(a/b) = \ln(a) - \ln(b)$ $\qquad$ $\ln(a^b) = b\ln(a)$