

Problem 1 on WeBWorK set 8.

Find the volume of the solid whose base is the region enclosed by  $y=x^2$  and  $y=1$ , and the cross sections perpendicular to the  $y$ -axis are squares.

Note that it talked about the cross section perpendicular to the  **$y$ -axis**, not the  $x$ -axis.

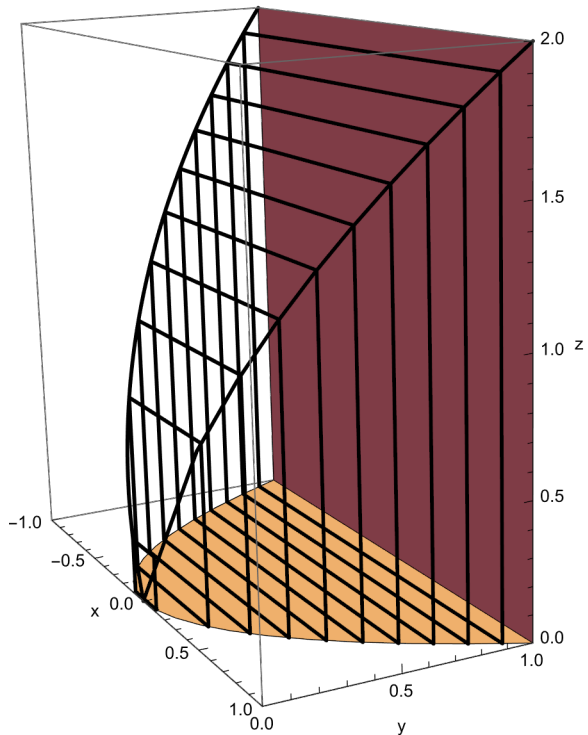
In this figure, the base is orange. The cross section at  $y = y_i$  is a square with side length  $2\sqrt{y_i}$ , so  $A(y_i) = 4y_i$  and the volume of the slice between  $y = y_i$  and  $y = y_i + \Delta y$  is  $\Delta V_i = 4y_i \Delta y$ . Adding the volume of all these slices, and taking the limit of  $\Delta y \rightarrow 0$  gives  $V = \int_0^1 4y \, dy = 2$

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In[ ]:= Graphics3D[ { (*base*) Polygon[Table[{x, x^2, 0}, {x, -1, 1, .1}]],
  (*face at y = 1 *) Polygon[{{1, 1, 0}, {1, 1, 2}, {-1, 1, 2}, {-1, 1, 0}}],
  (* square slices at y = constant *), Thick, Table[
    Line[{{ {sqrt[y], y, 0}, {sqrt[y], y, 2 sqrt[y]}, {-sqrt[y], y, 2 sqrt[y]}, {-sqrt[y], y, 0}, {sqrt[y], y, 0}}}],
    {y, .01, 1, .1} ],
  (* edge on right *)
  Line[Table[{sqrt[y], y, 2 sqrt[y]}, {y, 0, 1, .1}]],
  (* edge on left *)
  Line[Table[{-sqrt[y], y, 2 sqrt[y]}, {y, 0, 1, .01}]]],
  PlotRange -> {{-1, 1}, {0, 1}, {0, 2}}, Axes -> True, AxesLabel -> {"x", "y", "z"} ]

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Out[ ]:=



For 3 points of extra credit, find  $A(x)$  for this solid and compute the volume by integrating  $V =$

$$\int_{-1}^1 A(x) dx.$$

Submit a paper version of the computation to Prof. Swift by Friday, September 28 at the beginning of class.