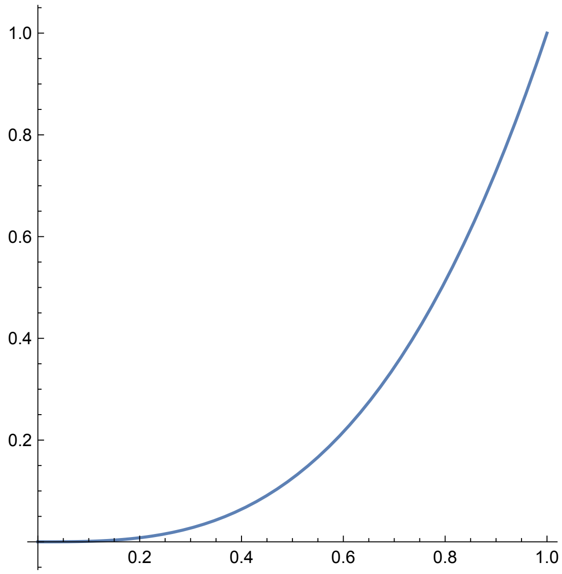
In[12]:=
```
f[x_] := x^3;
Plot[f[x], {x, 0, 1}, AspectRatio → Automatic]
```

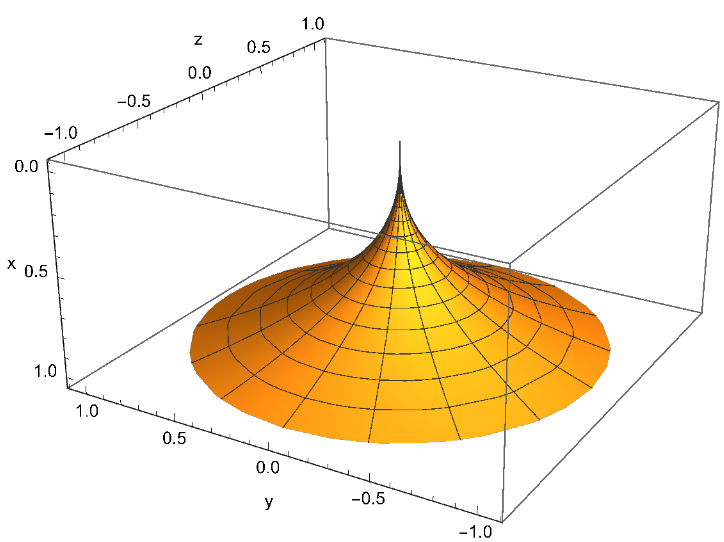
Out[13]=



In my YouTube video I computed the area of the surface when that curve is rotated about the x-axis. Here is a picture of that surface.

In[14]:=
```
ParametricPlot3D[{x, f[x] Cos[θ], f[x] Sin[θ]}, {x, 0, 1},
 {θ, 0, 2 π}, PlotRange → All, AxesLabel → {"x", "y", "z"}]
```

Out[14]=



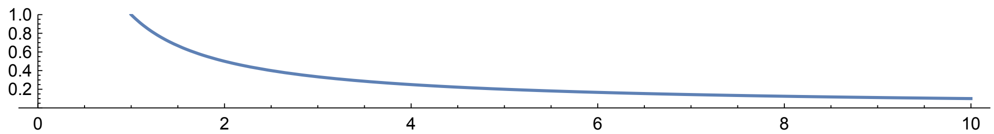
That point is very sharp! Suppose x = 1 represents one meter, so this object has height 1m and diameter 2m. Then $x = 10^{-3}$ is one millimeter from the tip. That is a distance we can easily see. The diameter of the surface at that height is $2*(10^{-3})^3$m = $2 * 10^{-9}$m. That is 2 nanometers in diameter, and a single gold atom is about a third of a nanometer in diameter.

As another example, consider a new function , and spin it around the x-axis.

In[15]:=
```
g[x_] := 1/x
```

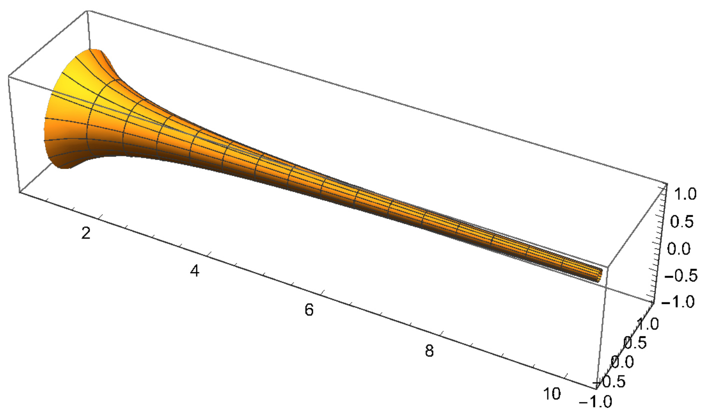
In[16]:=
```
Plot[g[x], {x, 1, 10}, AspectRatio → Automatic, PlotRange → {0, 1}]
```

Out[16]=



In[22]:=
```
ParametricPlot3D[{x, g[x] Cos[θ], g[x] Sin[θ]},
 {x, 1, 10}, {θ, 0, 2 π}, PlotRange → All]
```

Out[22]=



This looks like a trumpet, and it is called Gabriel's horn when it is extended to $x \longrightarrow \infty$. According to Wikipedia, the name refers to the Christian tradition where the archangel Gabriel blows the horn to announce Judgment Day.

Suppose that x goes from x = 1 to x = b. Then the area of the surface depends on b:

$$A(b) = \int_1^b 2\,\pi\, x^{-1} \sqrt{1 + \left(-x^{-2}\right)^2}\, dx = \int_1^b 2\,\pi\, \frac{\sqrt{1 + x^{-4}}}{x}\, dx$$

That integral is in fact "elementary" which is not to say "easy". The antiderivative can be written in terms of elementary functions. (Note the inverse of the hyperbolic tangent function.)

In[18]:=

$$\int 2\,\pi\, \frac{\sqrt{1 + x^{-4}}}{x}\, dx$$

Out[18]=

$$2\,\pi \left(-\frac{1}{2}\sqrt{1 + \frac{1}{x^4}} + \frac{\sqrt{1 + \frac{1}{x^4}}\; x^2\, \mathrm{ArcTanh}\left[\frac{x^2}{\sqrt{1+x^4}}\right]}{2\,\sqrt{1 + x^4}}\right)$$

That is too complicated to deal with. But note that we can compare that integral to one we can evaluate easily.

$$A(b) = \int_1^b 2\,\pi\, \frac{\sqrt{1 + x^{-4}}}{x}\, dx > 2\,\pi \int_1^b \frac{1}{x}\, dx = 2\pi \ln(b).$$

The next line shows that Mathematica use Log[ ] for the natural logarithm, and also it is allowing the possibility that b is a complex number.

In[19]:= $2\pi \int_1^b \frac{1}{x}\, dx$

Out[19]= $2\pi \operatorname{Log}[b]$ if $\operatorname{Re}[b] > 0 \,||\, b \notin \mathbb{R}$

By the comparison theorem for improper integrals, we see that the area of Gabriel's horn is infinite, since A(b) $\longrightarrow\infty$ as b$\longrightarrow\infty$.

However, the volume enclosed by part from x = 1 to x = b is V(b) = $\pi(1-\frac{1}{b})$

In[20]:= $\pi \int_1^b \left(\frac{1}{x}\right)^2 dx$

Out[20]= $\left(1-\frac{1}{b}\right)\pi$ if $\operatorname{Re}[b] > 0 \,||\, b \notin \mathbb{R}$

Note that V(b) $\longrightarrow \pi$ as b $\longrightarrow \infty$. Thus, Gabriel's horn only holds a finite amount of paint (3.14159... gallons) but the surface area is infinite, so there is not enough paint in the world to paint the outside of the surface! This is an interesting paradox.

The way I resolve the paradox is that when x is large enough, the horn is so thin that a paint molecule won't go any deeper into the hole. But when you paint the outside you are putting a fixed thickness of paint on the surface, which will be thousands of molecules thick.

You can search "NAU Mathematica" and find out how to get Mathematica installed on your own machine. Follow "Getting Mathematica" and "Student's personally owned machines"