Proper Application of Convergence Tests.

137 Paper Homework – Swift

A. Consider $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^3+1}}$.

$0 \le \frac{1}{\sqrt{n^3+1}} \le \frac{1}{\sqrt{n^3}} = \frac{1}{n^{3/2}}$ for all $n \ge 2$.

we know $\sum_{n=2}^{\infty} \frac{1}{n^{3/2}}$ converges, since it's a p-series, with $p = \frac{3}{2} > 1$.

Therefore, $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n^3+1}}$ converges by the comparison test.

B. was already posted and not part of the homework

C. Consider $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n}{1+n}$.

Note that the sequence of terms is $\left\{ \frac{+1}{2}, \frac{-2}{3}, \frac{+3}{4}, \frac{-4}{5}, \dots \right\}$

We see that $\lim_{n \to \infty} \frac{(-1)^{n-1} n}{1+n}$ DNE.

Therefore, $\sum_{n=1}^{\infty} \frac{(-1)^{n-1} n}{1+n}$ diverges by the test for divergence.

$\left\{\frac{1}{n^{3/2}}\right\}$ converges?
$\sum \frac{1}{n^{3/2}}$ converges?
what does this mean?

As with the previous homework, many people wrote "$\cancel{\frac{1}{n^{3/2}} \text{ converges}}$". Remember that every series has 2 sequences associated with it: the sequence of terms and the sequence of partial sums.

$\lim_{n\to\infty} \frac{1}{n^{3/2}} = 0$, or $\left\{\frac{1}{n^{3/2}}\right\}$ converges to 0. Also, $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ converges.

or $\frac{1}{n^{3/2}} \to 0$ as $n \to \infty$.