MAT 137 (Calculus II) Prof. Swift
Worksheet on the Area between Curves

In this worksheet we will solve this problem step-by-step.

Find the area of the region bounded by the curves $y = x^3 - x^2 - 2x$ and $y = 4x$.

1. Let $f(x) := x^3 - x^2 - 2x$. Factor $f(x)$. Also, compute $f'(x)$ and evaluate $f'(0)$. Make a rough sketch of the graph $y = f(x)$. (You do not need to find the critical points of f.) $f(x) = x^3 - x^2 - 2x = x(x^2 - x - 2) = x(x+1)(x-2)$

2. Put line $y = 4x$ on the same graph. $f'(x) = 3x^2 - 2x - 2$, $f'(0) = -2$

3. Find the x-values of the intersections between the two curves.

4. Write an expression for A, the area of the region bounded by the two curves as the sum of two definite integrals. You do not need to evaluate the integrals.

5. If you have time, use the internet to evaluate the integrals. You should get $A = \frac{253}{12} \approx 21.1$.

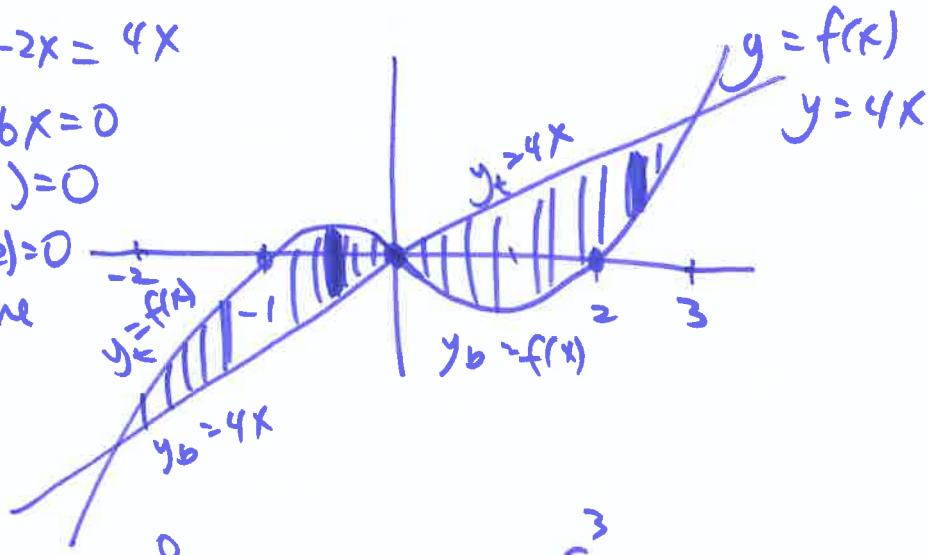
3. $x^3 - x^2 - 2x = 4x$

$x^3 - x^2 - 6x = 0$

$x(x^2 - x - 6) = 0$

$x(x-3)(x+2) = 0$

$x = 0, 3, -2$ are intersection x-values



$$A = \int_{-2}^{0} (f(x) - 4x)\,dx + \int_{0}^{3} 4x - f(x)\,dx$$

$$A = \int_{-2}^{0} x^3 - x^2 - 6x\,dx + \int_{0}^{3} -x^3 + x^2 + 6x\,dx$$