MAT 137 (Calculus II) Prof. Swift

Quiz/Worksheet on the Integral Test, the Comparison Test, and the Limit Comparison Test

You may work on this with other people, but turn in your own quiz.

1. Use the integral test to determine if $\sum_{n=1}^{\infty} e^{-n}$ converges.

$$\int_1^\infty e^{-x}\,dx = \lim_{b\to\infty}\int_1^b e^{-x}\,dx = \lim_{b\to\infty}\left[-e^{-x}\right]_1^b = \lim_{b\to\infty}\left(-e^{-b}+e^{-1}\right) = e^{-1}$$

Therefore, $\sum_{n=1}^{\infty} e^{-n}$ converges by the integral test.

2. Show that the series in problem 1 is a geometric series with $|r| < 1$. What does the series converge to?

$\sum_{n=1}^{\infty} e^{-n} = \sum_{n=1}^{\infty} (e^{-1})^n$, so the series is geometric with common ratio $r = e^{-1}$, and $|r| = e^{-1} \approx \frac{1}{3} < 1$.

First term is e^{-1}, so $\sum_{n=1}^{\infty} e^{-n} = \frac{e^{-1}}{1-e^{-1}} = \boxed{\frac{1}{e-1}}$.

For the next three problems, compare (or limit compare) the series to $\sum_{n=1}^{\infty} \frac{1}{n^2}$, or $\sum_{n=2}^{\infty} \frac{1}{n^2}$, which converge since they are p series with $p = 2 > 1$.

3. Use the Comparison Test to determine if $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converges.

$0 < \frac{1}{n^2+1} \leq \frac{1}{n^2}$, and $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges, so $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converges.

4. Can you determine if $\sum_{n=2}^{\infty} \frac{1}{n^2-1}$ converges using the Comparison Test?

NO. $\frac{1}{n^2-1} \geq \frac{1}{n^2} \geq 0$. The comparison test is inconclusive. (Larger than convergent proves nothing.)
and $\sum_{n=2}^{\infty} \frac{1}{n^2}$ converges.

5. Use the Limit Comparison Test to determine if $\sum_{n=2}^{\infty} \frac{1}{n^2-1}$ converges.

$\lim_{n\to\infty} \frac{\frac{1}{n^2-1}}{\frac{1}{n^2}} = \lim_{n\to\infty} \frac{n^2}{n^2-1} \overset{L'H}{=} \lim_{n\to\infty} \frac{2n}{2n} = 1$. $\sum_{n=2}^{\infty} \frac{1}{n^2}$ Converges.

So $\sum_{n=2}^{\infty} \frac{1}{n^2-1}$ converges by the limit comparison test.

Nonsense things you cannot write.

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1. Use the integral test to determine if $\sum_{n=1}^{\infty} e^{-n}$ converges.

$\int_1^{\infty} e^{-n}\,dn$, $\int_1^{\infty} e^{-x}\,dx = \left[-e^{-x}\right]_1^{\infty} = -e^{-\infty} + e^{-1}$

n is an integer $\qquad$ $e^{-\infty}$ is undefined. Use limits.

2. Show that the series in problem 1 is a geometric series with $|r| < 1$. What does the series converge to?

$r < 1$. This means Nothing regarding convergence

$r = -2$ satisfies $r < 1$, and $\sum_{n=0}^{\infty} (-2)^n$ diverges.

Also, be sure to Show what is asked

More Nonsense

For the next three problems, compare (or limit compare) the series to $\sum_{n=1}^{\infty} \frac{1}{n^2}$, or $\sum_{n=2}^{\infty} \frac{1}{n^2}$, which converge since they are p series with $p = 2 > 1$.

3. Use the Comparison Test to determine if $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converges.

$\frac{1}{n^2+1}$ converges. Say $\lim_{n\to\infty} \frac{1}{n^2+1} = 0$, or $\left\{\frac{1}{n^2+1}\right\}_{n=1}^{\infty}$ converges or $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$ converges.

4. Can you determine if $\sum_{n=2}^{\infty} \frac{1}{n^2-1}$ converges using the Comparison Test?

"No" is NOT enough. It must be explained.

$\sum_{n=2}^{\infty} \frac{1}{n^2-1} > \frac{1}{n^2}$ ~~Test~~ $\sum_{n=2}^{\infty} \frac{1}{n^2-1}$ is a number. $\frac{1}{n^2}$ is a function of n. Not the same.

5. Use the Limit Comparison Test to determine if $\sum_{n=2}^{\infty} \frac{1}{n^2-1}$ converges.

$\sum_{n=2}^{\infty} \frac{1}{n^2-1}$ converges to ∞. $\frac{n^2}{n^2-1} = 1$

It's OK to say $\frac{n^2}{n^2-1} \to 1$ as $n \to \infty$, or $\lim_{n\to\infty} \frac{n^2}{n^2-1} = 1$.