MAT 137 (Calculus II) Prof. Swift

In-class worksheet: Review of Integration

I put 4 review problems on the board today:

1. Evaluate $\int \sqrt{x}(x^2+1)dx$

2. Evaluate $\int_0^{\sqrt{\pi}} x\sin(x^2)dx$

3. Evaluate $\int \frac{3+x}{1+x^2}dx$

4. Evaluate $\int x\sqrt{1+x}dx$

The solutions are on the next page:

Solutions:

1. $\displaystyle\int \sqrt{x}(x^2+1)dx = \int x^{5/2} + x^{1/2}\,dx = \frac{x^{7/2}}{7/2} + \frac{x^{3/2}}{3/2} + C = \tfrac{2}{7}x^{7/2} + \tfrac{2}{3}x^{3/2} + C$

2. Method A: Start with the indefinite integral, with $u = x^2$ substitution (So $du = 2xdx$). This finds an antiderivative of $x\sin(x^2)$.

$$\int x\sin(x^2)dx = \tfrac{1}{2}\int \sin(u)du = -\tfrac{1}{2}\cos(u) + C = -\tfrac{1}{2}\cos(x^2) + C.$$

Then, evaluate the definite integral using the Fundamental Theorem of Calc, Part I.

$$\int_0^{\sqrt{\pi}} x\sin(x^2)dx = -\frac{1}{2}\cos(x^2)\Big|_0^{\sqrt{\pi}} = -\frac{1}{2}\cos((\sqrt{\pi})^2) + \frac{1}{2}\cos(0^2) = -\frac{1}{2}\cos(\pi) + \frac{1}{2}\cos(0) = -\frac{1}{2}(-1) + \frac{1}{2}(1) = \frac{1}{2} + \frac{1}{2} = 1.$$

2. Method B: Do the same substitution, $u = x^2$, and change the lower limit from $x = 0$ to $u = 0^2 = 0$, and change the upper limit from $x = \sqrt{\pi}$ to $u = (\sqrt{\pi})^2 = \pi$.

$$\int_0^{\sqrt{\pi}} x\sin(x^2)dx = \frac{1}{2}\int_0^{\pi} \sin(u)du = -\frac{1}{2}\cos(u)\Big|_0^{\pi} = -\frac{1}{2}\cos(\pi) + \frac{1}{2}\cos(0) = -\frac{1}{2}(-1) + \frac{1}{2}(1) = \frac{1}{2} + \frac{1}{2} = 1.$$

3. The trick is to separate into two integrals. Solve the first by inspection, and the second one with the substitution $u = 1 + x^2$ (so $du = 2xdx$).

$$\int \frac{3+x}{1+x^2}dx = \int \frac{3}{1+x^2}dx + \int \frac{x}{1+x^2}dx = 3\arctan(x) + \frac{1}{2}\int \frac{du}{u} = 3\arctan(x) + \tfrac{1}{2}\ln|u| + C = 3\arctan(x) + \tfrac{1}{2}\ln|1+x^2| + C = 3\arctan(x) + \tfrac{1}{2}\ln(1+x^2) + C.$$

4. For this one, $u = 1 + x$ works. Note that $du = dx$, and $x = u - 1$.

$$\int x\sqrt{1+x}dx = \int (u-1)\sqrt{u}\,du = \int u^{3/2} - u^{1/2}du = \frac{u^{5/2}}{5/2} - \frac{u^{3/2}}{3/2} + C = \frac{2}{5}u^{5/2} - \frac{2}{3}u^{3/2} + C = \frac{2}{5}(1+x)^{5/2} - \frac{2}{3}(1+x)^{3/2} + C.$$