MAT 137 (Calculus I) Prof. Swift

In-class worksheet: Integration by Parts

Evaluate these integrals using Integration by Parts. The rule of thumb, LIATE, works for these examples.

1. Evaluate $\int \overbrace{x}^{\text{T}} \overbrace{\sin(2x)}^{\text{A}}\, dx$ $= x\left(-\frac{1}{2}\cos(2x)\right) - \int \left(-\frac{1}{2}\right)\cos(2x)\,dx$ $= -\frac{1}{2}x\cos(2x) + \frac{1}{4}\sin(2x) + C$

$u = x$ $\quad dv = \sin(2x)\,dx$

$du = dx$ $\quad v = \int \sin(2x)\,dx = -\frac{1}{2}\cos(2x)$, by inspection, or substitution $u = 2x$.

2. Evaluate $\int \underbrace{\arctan(x)}_{\text{I}}\, dx$

$u = \arctan(x)$ $\quad dv = dx$

$du = \frac{1}{1+x^2}\,dx$ $\quad v = x$

$u = 1+x^2$, $du = 2x\,dx$

So $\int \arctan(x)\,dx = \arctan(x)\cdot x - \int x \cdot \frac{1}{1+x^2}\,dx$

$= x\arctan(x) - \frac{1}{2}\int \frac{1}{u}\,du$

$= x\arctan(x) - \frac{1}{2}\ln|u| + C$

$= x\arctan(x) - \frac{1}{2}\ln(1+x^2) + C$

$= x\arctan(x) - \ln\left(\sqrt{1+x^2}\right) + C$

either form is OK. Note that $1+x^2 > 0$, so $|1+x^2| = 1+x^2$ (the absolute value is not needed.)