MAT 137 (Calculus II) Prof. Swift

In-class worksheet: Trig

1. Evaluate $\int \sin^3(x)\cos^2(x)\,dx$ = $\int (1-u^2)\,u^2\,(-du) = \int u^4 - u^2\,du = \frac{u^5}{5} - \frac{u^3}{3} + C$

odd

$u = \cos(x)$, $\sin^2(x) = 1 - \cos^2(x) = 1 - u^2$

$du = -\sin(x)\,dx$

$= \boxed{\frac{\cos^5(x)}{5} - \frac{\cos^3(x)}{3} + C}$

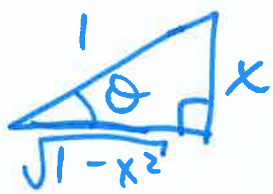
2. Evaluate $\int \sqrt{1-x^2}\,dx$.

Hint: $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$ is needed at the end of the calculation.

Draw right Δ with sides $1, x, \sqrt{1-x^2}$

$(\text{sides})^2$ $1, x^2, 1-x^2$

$(1-x^2) + (x^2) = 1$



Let $x = \sin\theta$, so $dx = \cos(\theta)\,d\theta$

Note: $\sqrt{1-x^2} = \cos\theta$

$$\int \sqrt{1-x^2}\,dx = \int \cos(\theta)\cdot\cos(\theta)\,d\theta = \int \cos^2(\theta)\,d\theta$$

$$= \int \tfrac{1}{2}(1+\cos(2\theta))\,d\theta$$

$$= \tfrac{1}{2}\theta + \tfrac{1}{4}\sin(2\theta) + C$$

To write the answer in terms of x, ~~~~ note that ~~~~ $\theta = \arcsin(x)$

so $\int \sqrt{1-x^2}\,dx = \boxed{\tfrac{1}{2}\arcsin(x) + \tfrac{1}{4}\sin(2\arcsin(x)) + C}$

This is OK, but it's better to use the trig identity

$\sin(2\theta) = 2\sin(\theta)\cos(\theta) = 2x\sqrt{1-x^2}$

so $\boxed{\int \sqrt{1-x^2}\,dx = \tfrac{1}{2}\arcsin(x) + \tfrac{1}{2}x\sqrt{1-x^2} + C}$