

MAT 137 (Calculus II) Prof. Swift
In-class worksheet: Partial Fractions

Evaluate $\int \frac{2x+11}{x^2-x-2} dx$ using partial fractions.

$$x^2 - x - 2 = (x+1)(x-2)$$

$$\text{so } \frac{2x+11}{x^2-x-2} = \frac{2x+11}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2} \text{ for some } A \text{ and } B.$$

multiply both sides by $(x+1)(x-2)$

$$2x+11 = A \overset{(x-2)}{\cancel{(x+1)}} + B(x+1) \leftarrow \text{is true for all } x \text{ if } A \text{ \& } B \text{ are chosen correctly.}$$

$$\text{let } x = -1: 2(-1)+11 = A(-1-2) + B(-1+1)$$

$$9 = \cancel{0} - 3A + B \cdot 0$$

$$A = \frac{9}{-3} = -3$$

$$\text{let } x = 2: 2 \cdot 2 + 11 = A(2-2) + B(2+1)$$

$$15 = A \cdot 0 + 3B$$

$$B = \frac{15}{3} = 5$$

Therefore,

$$\int \frac{2x+11}{x^2-x-2} dx = \int \frac{-3}{x+1} + \frac{5}{x-2} dx$$

$$= -3 \int \frac{1}{x+1} dx + 5 \int \frac{1}{x-2} dx$$

$$= \boxed{-3 \ln|x+1| + 5 \ln|x-2| + C}$$