MAT 137 (Calculus II) Prof. Swift
In-class worksheet: Improper integrals

1. Write the following improper integral as the sum of the limits of 2 different definite integrals. Do not attempt to evaluate the definite integrals.

$$\int_0^\infty \frac{1}{\sqrt{x+x^3}}\,dx. = \lim_{a \to 0^+} \int_a^1 \frac{1}{\sqrt{x+x^2}}\,dx + \lim_{b \to \infty} \int_1^b \frac{1}{\sqrt{x+x^3}}\,dx$$

2. Write the following improper integral as the limit of a definite integral. Then find the value of the improper integral, or show that the improper integral is divergent.

$$\int_0^\infty \frac{1}{1+x^2}\,dx = \lim_{b \to \infty} \int_0^b \frac{1}{1+x^2}\,dx = \lim_{b \to \infty} \arctan(x)\Big|_0^b$$

$$= \lim_{b \to \infty} \left(\arctan(b) - \arctan(0) \right) \qquad \overset{\nearrow 0}{}$$

$$= \lim_{b \to \infty} \arctan(b) = \boxed{\frac{\pi}{2}}$$

$$y = \arctan(x).$$