MAT 137 (Calculus II) Prof. Swift

In-class worksheet: Comparison Theorem for Improper Integrals

1. Evaluate the improper integral. Hint: Start by writing " = lim".

$$\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx = \lim_{t\to 1^-} \int_0^t \frac{1}{\sqrt{1-x^2}}\,dx = \lim_{t\to 1^-} \arcsin(x)\Big|_0^t = \lim_{t\to 1^-} \arcsin(t) = \boxed{\frac{\pi}{2}}$$

Try to determine if the improper integral converges or diverges, using the comparison theorem. In each case, compare to $\int_1^\infty \frac{1}{x^p}\,dx$ for some value of p.

2. $\int_1^\infty \frac{1}{\sqrt{x^3+1}}\,dx \le \int_1^\infty \frac{1}{\sqrt{x^3+0}}\,dx = \int_1^\infty \frac{1}{x^{3/2}}\,dx$, which converges, since $p = \frac{3}{2} > 1$.

$\boxed{\text{Therefore, } \int_1^\infty \frac{1}{\sqrt{x^3+1}}\,dx \text{ converges}}$

3. $\int_1^\infty \frac{\sin^2(x)}{x}\,dx \le \int_1^\infty \frac{1}{x}\,dx$, which diverges. $(p=1)$

$\boxed{\text{The comparison test is inconclusive}}$

4. $\int_1^\infty \frac{1+\cos^2(x)}{\sqrt{x}}\,dx \ge \int_1^\infty \frac{1}{\sqrt{x}}\,dx = \int_1^\infty \frac{1}{x^{\frac{1}{2}}}\,dx$, which diverges since $p = \frac{1}{2} \le 1$.

$\boxed{\text{Therefore, } \int_1^\infty \frac{1+\cos^2(x)}{\sqrt{x}}\,dx \text{ diverges.}}$