MAT 137 (Calculus II) Prof. Swift

In-class worksheet: Integration Techniques

Evaluate the indefinite integrals

1. $\int \frac{x^2+3x+2}{x}\,dx$ $= \int x+3+\frac{2}{x}\,dx = \boxed{\frac{x^2}{2}+3x+2\ln|x|+C}$

2. $\int e^x+2\sin(x)+3\cos(x)+\frac{1}{1+x^2}+\frac{2}{\sqrt{1-x^2}}\,dx$ $= \boxed{e^x-2\cos(x)+3\sin(x)+\arctan(x)+2\arcsin(x)+C}$

3. $\int xe^x\,dx$ $= xe^x-\int e^x dx = \boxed{xe^x-e^x+C}$

$u=x \quad dv=e^x dx$

$du=dx \quad v=e^x$

4. $\int \frac{x}{x^2-x-2}\,dx$ $= \int \frac{\frac{1}{3}}{x+1}+\frac{\frac{2}{3}}{x-2}\,dx = \boxed{\frac{1}{3}\ln|x+1|+\frac{2}{3}\ln|x-2|+C}$

$\frac{x}{x^2-x-2} = \frac{x}{(x+1)(x-2)} = \frac{A}{x+1}+\frac{B}{x-2}$, where $x=A(x-2)+B(x+1)$

$x=1 \Rightarrow 1=A(-1-2)+B\cdot 0 \therefore A=\frac{1}{3}$

$x=2 \Rightarrow 2=A(2-2)+B(2+1) \therefore B=\frac{2}{3}$

5. $\int \frac{2x-1}{x^2-x-2}\,dx$ $= \int \frac{1}{u}\,du = \ln|u|+C = \boxed{\ln|x^2-x-2|+C}$

let $u=x^2-x-2$, $du=(2x-1)dx$

Use trig substitution to rewrite the following integral as an integral in θ, without any square roots, but do not evaluate the integral.

any of these gets full credit

6. $\int \frac{\sqrt{1-x^2}}{x}\,dx$ $= \boxed{\int \frac{\cos(\theta)}{\sin(\theta)}\cos(\theta)\,d\theta = \int \frac{1}{\tan(\theta)}\cos(\theta)\,d\theta = \int \frac{\cos^2(\theta)}{\sin(\theta)}\,d\theta}$

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$\sin(\theta)=x$, so $dx=\cos(\theta)\,d\theta$

$\cos(\theta)=\sqrt{1-x^2}$

$\tan(\theta)=\frac{x}{\sqrt{1-x^2}}$

$(\sqrt{1-x^2})^2+x^2 \stackrel{?}{=} 1$

$1-x^2+x^2 \stackrel{\checkmark}{=} 1$

Note: we could do 5 by partial fractions.

you find $\frac{2x-1}{x^2-x-2} = \frac{1}{x+1}+\frac{1}{x-2}$. So (skipping steps),

$\int \frac{2x-1}{x^2-x-2}\,dx = \int \frac{1}{x+1}+\frac{1}{x-2}\,dx = \boxed{\ln|x+1|+\ln|x-2|+C}$

An identity with logarithms shows this is the same.