

MAT 137 (Calculus II) Prof. Swift
In-class worksheet: Integration Techniques

Evaluate the indefinite integrals

1. $\int \frac{x^2 + 3x + 2}{x} dx = \int x + 3 + \frac{2}{x} dx = \frac{x^2}{2} + 3x + 2 \ln|x| + C$

2. $\int e^x + 2 \sin(x) + 3 \cos(x) + \frac{1}{1+x^2} + \frac{2}{\sqrt{1-x^2}} dx = e^x - 2 \cos(x) + 3 \sin(x) + \arctan(x) + 2 \arcsin(x) + C$

3. $\int x e^x dx = x e^x - \int e^x dx = x e^x - e^x + C$
Handwritten notes: $y = x, dv = e^x dx, du = dx, v = e^x$

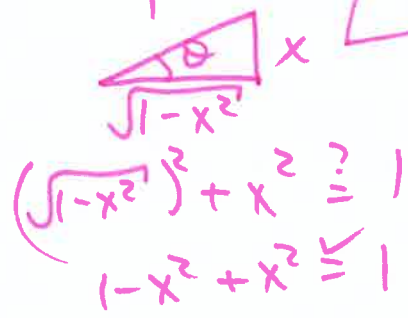
4. $\int \frac{x}{x^2 - x - 2} dx = \int \frac{\frac{1}{3}}{x+1} + \frac{\frac{2}{3}}{x-2} dx = \frac{1}{3} \ln|x+1| + \frac{2}{3} \ln|x-2| + C$

$\frac{x}{x^2 - x - 2} = \frac{x}{(x+1)(x-2)} = \frac{A}{x+1} + \frac{B}{x-2}$, where $x = A(x-2) + B(x+1)$
 $x=1 \Rightarrow 1 = A(-1-2) + B \cdot 0 \dots \therefore A = -\frac{1}{3}$
 $x=2 \Rightarrow 2 = A(2-2) + B(2+1) \dots \therefore B = \frac{2}{3}$

5. $\int \frac{2x-1}{x^2-x-2} dx = \int \frac{1}{u} du = \ln|u| + C = \ln|x^2-x-2| + C$
Handwritten notes: $u = x^2 - x - 2, du = (2x-1) dx$

Use trig substitution to rewrite the following integral as an integral in θ , without any square roots, but do not evaluate the integral.

6. $\int \frac{\sqrt{1-x^2}}{x} dx = \int \frac{\cos(\theta)}{\sin(\theta)} \cos(\theta) d\theta = \int \frac{1}{\tan(\theta)} \cos(\theta) d\theta = \int \frac{\cos^2(\theta)}{\sin(\theta)} d\theta$



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 $\sin(\theta) = x, \text{ so } dx = \cos(\theta) d\theta$
 $\cos(\theta) = \sqrt{1-x^2}$
 $\tan(\theta) = \frac{x}{\sqrt{1-x^2}}$

Note: we could do 5 by partial fractions.
 you find $\frac{2x-1}{x^2-x-2} = \frac{1}{x+1} + \frac{1}{x-2}$. SO (skipping steps),

$\int \frac{2x-1}{x^2-x-2} dx = \int \frac{1}{x+1} + \frac{1}{x-2} dx = \ln|x+1| + \ln|x-2| + C$

An identity with logarithms shows this is the same.