MAT 137 (Calculus II) Prof. Swift

In-class worksheet:Area Between Curves

or $x = y+2$

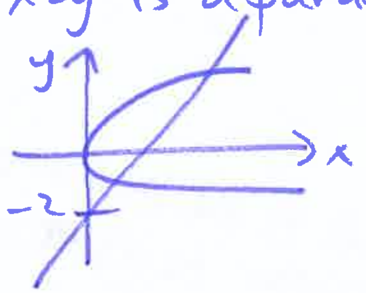
1. Make a rough sketch of the region bounded by the curves $x = y^2$ and $y = x - 2$.

2. Compute the points where the curves intersect, and make a more accurate sketch.

3. Set up a definite integral for the area of the region. (Decide whether to integrate with respect to x or y.) You do *not* need to evaluate the integral.

4. To demonstrate how good your choice was, find an expression for the area as the sum of two definite integrals with respect to the other variable.

1. $x=y^2$ is a parabola that opens right



2. Set the two ~~values~~ expressions for x equal to get intersection

$y^2 = y+2$, or $y^2 - y - 2 = 0$

factor: $(y-2)(y+1) = 0$

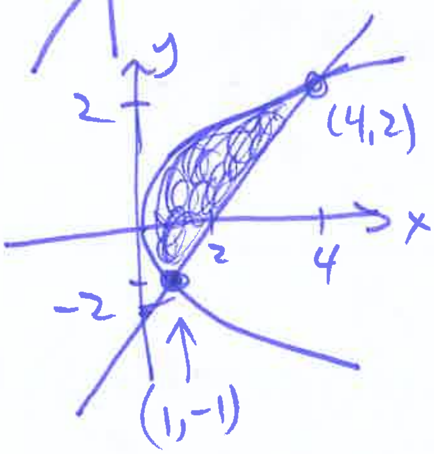
$y = 2$ and $y = -1$

The point has $x = y^2$, so the 2 points of intersection are

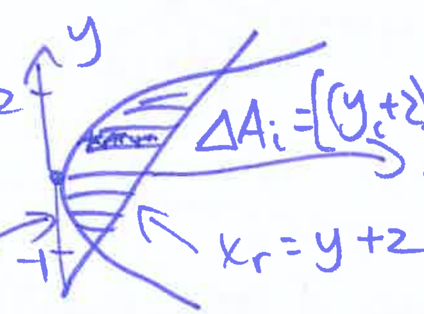
$(2^2, 2) = (4,2)$ and $((-1)^2, -1) = (1,-1)$

~~2.~~

2.



3.



$\Delta A_i = [(y_i+2) - y_i^2]\Delta y$

$A \approx \sum_{i=1} [(y_i+2) - y_i^2] \Delta y$

~~$A = \int (y_i + 2) -$~~

$y_0 = -1$ is first slice

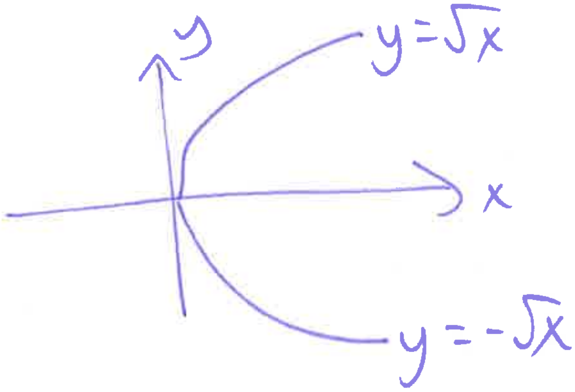
$y_n = 2$ is last slice.

This gives the limits of integration

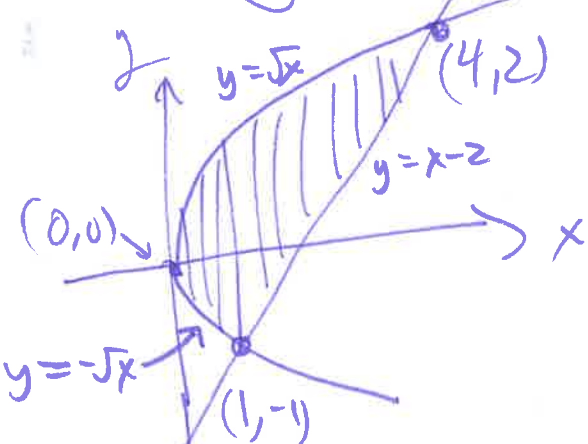
$$\boxed{A = \int_{-1}^{2} (\underbrace{y+2}_{x_{right}} - \underbrace{y^2}_{x_{left}})\,dy}$$

4. Now get an x-integral.

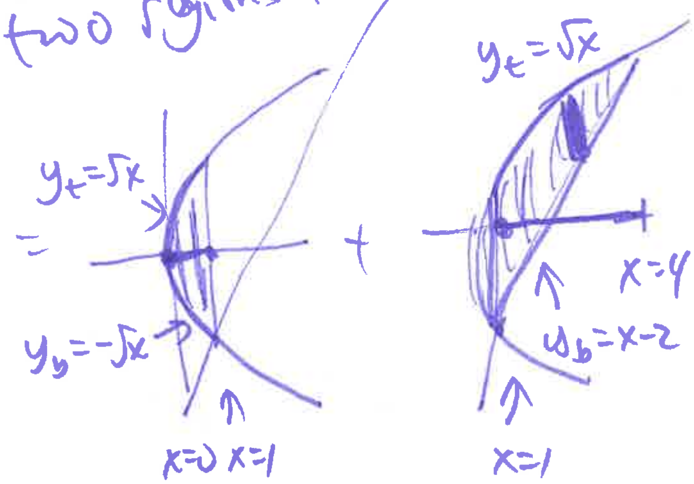
$x = y^2$ has two parts, $y = \sqrt{x}$ and $y = -\sqrt{x}$



Redraw the region



This needs to be broken into two regions to do the integrals.



$$\boxed{A = \int_0^1 \Big(\underset{\substack{\uparrow \\ y_{top}}}{\sqrt{x}} - \underset{\substack{\uparrow \\ y_{bottom}}}{(-\sqrt{x})}\Big)\,dx + \int_1^4 \Big(\underset{\substack{\uparrow \\ y_{top}}}{\sqrt{x}} - \underset{\substack{\uparrow \\ y_{bottom}}}{(x-2)}\Big)\,dx}$$

or $A = \int_0^1 2\sqrt{x}\,dx + \int_1^4 (\sqrt{x} - x + 2)\,dx$

Both expressions give $A = \frac{9}{2}$.