MAT 137 (Calculus II) Prof. Swift

In-class worksheet: Computing Volume of Solids by Parallel Slices

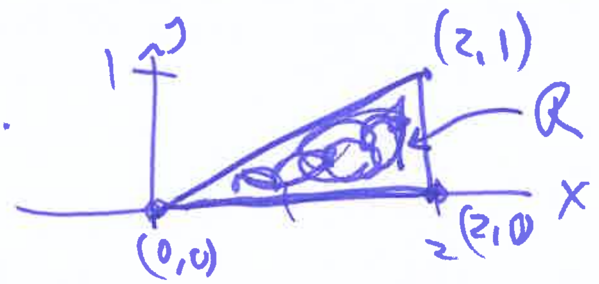
Let $\mathcal{R}$ be the triangle in the x-y plane with vertices $(0,0)$, $(2,0)$, and $(2,1)$.

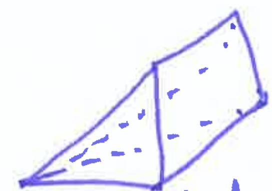
1. Sketch the region $\mathcal{R}$.

2. Find the volume of the solid whose base is $\mathcal{R}$, and the cross sections perpendicular to the x axis are squares.

3. Find the volume of the solid obtained by rotating the region $\mathcal{R}$ about the x-axis.

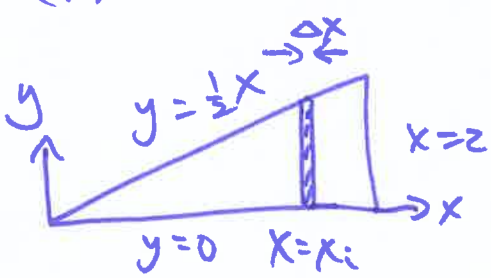
1.

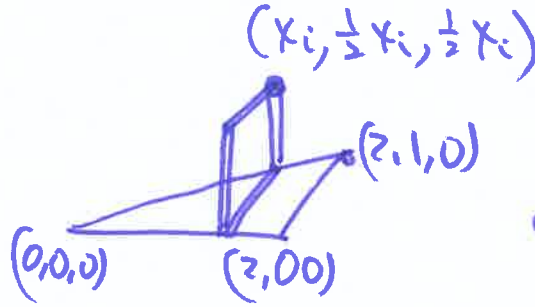


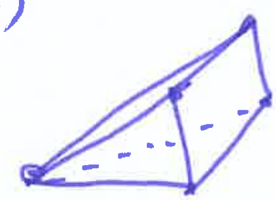


solid in 2.

2.

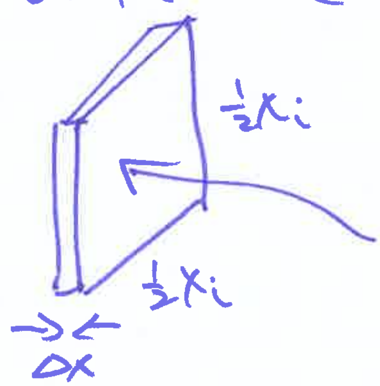






Solid in 2.

Let ΔV_i be the volume of the slice between $x=x_i$ and $x = x_i + \Delta x$.



$$\Delta V_i = \left(\tfrac12 x_i\right)\left(\tfrac12 x_i\right)\Delta x$$
$$= A(x_i)\,\Delta x$$
$$= \tfrac14 x_i^2\,\Delta x$$

$A(x_i) = \left(\tfrac12 x_i\right)^2 = \tfrac14 x_i^2$ is Area of the cross section at $x = x_i$

where $x_0 = 0$, $x_n = 2$

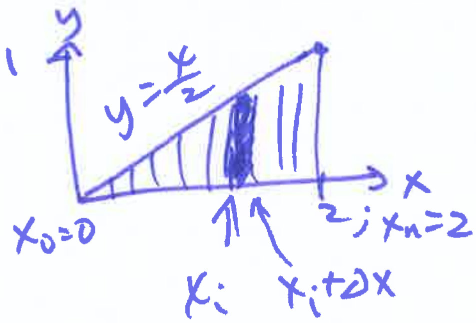
The volume of the n slices is

$$\sum_{i=0}^{n-1} \Delta V_i = \sum_{i=0}^{n-1} \tfrac14 x_i^2\,\Delta x \to \boxed{V = \int_0^2 \tfrac14 x^2\,dx}$$ as $n \to \infty$ (or $\Delta x \to 0$

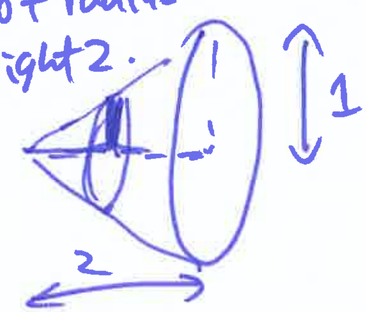
Computing the integral, $V = \frac{x^3}{4\cdot 3}\Big|_0^2 = \frac{2^3}{4\cdot 3} - \frac{0^3}{4\cdot 3} = \frac{8}{4\cdot 3} = \boxed{\frac{2}{3}}$

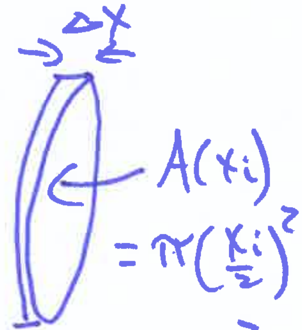
<u>Corrected!</u>

3. when the rectangle between $x = x_i$ and $x = x_i + \Delta x$ gets rotated about the x-axis, you get a pancake with volume ΔV_i.



Cone of radius 1 and height 2.





$= \pi \left(\frac{x_i}{2}\right)^2$

$= \pi \frac{x_i^2}{4}$

$$\Delta V_i = A(x_i)\Delta x = \pi \frac{x_i^2}{4} \Delta x$$

$$V \approx \sum_{i=0}^{n-1} A(x_i)\Delta x = \sum_{i=0}^{n-1} \pi \frac{x_i^2}{4} \Delta x.$$ Take limit $n \to \infty$ (or $\Delta x \to 0$)

$$\boxed{V = \int_0^2 \pi \frac{x^2}{4} dx}$$

Now evaluate integral.

$$V = \frac{\pi}{4}\int_0^2 x^2 dx = \frac{\pi}{4} \frac{x^3}{3}\Big|_0^2 = \frac{\pi}{4}\frac{2^3}{3} - 0 = \frac{\pi}{4}\cdot\frac{8}{3}$$

$$\boxed{V = \frac{2\pi}{3}}$$

Note that $V_{cone} = \frac{1}{3} V_{cylinder}$, where $V_{cylinder}$ is the volume of a cylinder of radius 1 and height 2.

$R = 1$

$H = 2$

$V_{cylinder} = \pi R^2 H$

$V_{cone} = \frac{1}{3}\pi R^2 H.$

$V_{cylinder} = 2\pi$

$V_{cone} = \frac{2\pi}{3}.$